A Snake for Model-Based Segmentation

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Abstract

Despite the promising results of numerous applications, the hitherto proposed snake techniques share some common problems: snake attraction by spurious edge points, snake degeneration (shrinking and flattening), convergence and stability of the deformation process, snake initialization and local determination of the parameters of elasticity. We argue here that these problems can be solved only when all the snake aspects are considered. The snakes proposed here implement a new potential field and external force in order to provide a deformation convergence, attraction by both near and far edges as well as snake behaviour selective according to the edge orientation. Furthermore, we conclude that in the case of model-based segmentation, the internal force should include structural information about the expected snake shape. Experiments using this kind of snakes for segmenting bones in complex hand radiographs show a significant improvement.

Keywords: snakes; elastic matching; model-based segmentation

1 Introduction

Snakes, introduced by Kass et al. in [8] provide a global solution to the segmentation problem. The main advantage is that they allow to integrate an initial contour estimation and to overcome several photometric abnormalities (contour gaps, hidden contours or edge points due to noise and texture). Matching using elastic curves has a very important aspect regarding model-based segmentation: they are able to locate and recognize objects from approximate models.

A snake is an elastic curve placed on an image that begins to deformate itself from an initial shape in order to adjust to the image features. The deformation is a result of the action of external forces that attract the snake towards image features and internal forces that keep smooth the shape of the curve. The solution is given by a minimum of the snake total energy.

A common problem in the different snakes techniques is how to avoid local minima of the energy functional [1, 4, 5, 8, 10]. Related to this problem different hierarchical frameworks are proposed based on scale space [10], on different image resolutions [2, 6] and on different thresholdings of the image edge map [14]. The snakes we consider [14] deform in a multithresholding deformation scheme similarly to a mould of a plasticine figure. Firstly, the snake is deformed in accordance with the more salient image features and afterwards, it is tuned according to the remaining image features. Thus, many insignificant edge points normally due to noise or other artefacts are avoided.

The local minima are due to the snake attraction by edge points that do not correspond with the object contour. The coarse-to-fine approach allows the snake to ignore the weak edge points, but still a need for more information appears in order to distinguish the spurious edges in the image. Therefore the potential of these snakes is function of both the magnitude and direction of the image gradient. This potential allows the snake to foresee the characteristics of the closest edge point and to manifest selective behaviour. The importance of contour direction information was also observed by Fua et al. [5], who used it in order to verify the result of the snake deformation. The difference is that the snake considered here does not begin to deform towards non suitable edge points. This is an important aspect because of the irreversible nature of the snake deformation. The external force that we propose undertakes to attract the snake only to suitable edge points. Besides, it distinguishes close and far edges and assures convergence and stability of the deformation process. Thus, the snakes are able to segment an object even in case of other close objects and initialization not very close to the solution, a situation in which most of the known snake techniques fail, attracted by contours of different objects.

Another property of the snakes we propose is that they can cope with the problem of the snake initialization. The multithresholding scheme of deformation allows the snake to see strong edges behind weaker ones [14]. The internal force of our snakes which preserves the structure of the object model makes the snake able to do some displacement when only some part of the snake is attracted.

The degeneration of the original snake is another important problem. In its attempt to smooth the curve, it is easy for the internal force to shrink the snake in a point. Another kind of degeneration, less mentioned in the bibliography, is the snake flattening in case of scarce edge segments. A well-known way to avoid the shrinking effect is to use Cohen's pressure force that simulates a balloon inflation. This gives rise to the problem of how to determine the parameters that assure the tolerance between the pressure force and the internal force. In [10, 17] the authors come back to the snake precursors, the deformable models

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[15]. There the internal energy contains the "natural" arc-length and curvature of the object in its natural state, used to prescribe the "desirable" spatial-step and curvature between the snaxels [10]. The snakes we consider use an internal energy that preserves the shape features of the object model avoiding in this way the degeneration effect.

Another effect is that such internal energy frees us from the local control problem. Recently, this problem was approached by using finite elements method with variable number and/or distance between the control points [7, 11]. In our case, as object's structure information is incorporated, the internal energy allows corners, more pronounced concavities and convexities in the neighbourhood where they are expected. Thus the parameters of the elasticity and rigidity remain global and the computational cost of the implementation keeps low. Finally, we show here that the use of this kind of internal force conserves all the properties of the numerical solution of the classical snake, which is an item reported as one of the main implementation difficulties for the local parameters of elasticity [10].

The article is organized as following: in section 2 we expose the fundamentals of 2D snakes, in section 3 and 4 the potential field of our snake and the snake external energy are discussed. Section 5 is dedicated to the internal energy. The results of applying the snakes to hand radiographs segmentation are discussed in section 6. Finally, conclusions are reported.

2 Snake Model: Fundamentals

A snake is a continuous curve that, from an initial state, tries to position itself dynamically on image features (f.e. edge points). It is deformed as a result of the influence of local forces derived from edge points, while this deformation remains smooth due to the effect of internal forces. The sum of the membrane energy, expressing the snake stretching, and of the thin-plate energy, expressing the snake bending, gives the internal snake energy:

$$E_{int}(u(s)) = \alpha(s)|u_s(s)|^2 + \beta(s)|u_{ss}(s)|^2$$

where $u(s) = (x(s), y(s))$ is the snake curve and s is the curve arc-length [8]. The parameters of elasticity α and β control the smoothness of the snake curve.

External forces help to push the snake towards the image features we are looking for. These forces are associated to a potential $P(x, y)$ which, in general, is defined in terms of gradient module of the image convolved by a Gaussian function [8]:

$$P(x, y) = -|\nabla (G(x, y) * I(x, y))|$$

or as a distance map of the edge points [3]:

$$P(x, y) = d(x, y), \quad P(x, y) = -e^{-d(x,y)^2}$$

where $d(x, y)$ denotes the distance between the pixel (x, y) and its closest edge point. The snake is moved by potential forces and tries to fall in a valley as if it was under the effect of gravity.

The total snake energy is given by the functional of the energies sum:

$$E_{snake} = \int_0^1 E_{int} + E_{ext} ds =$$

$$\int_0^1 \alpha(s)|u_s(s)|^2 + \beta(s)|u_{ss}(s)|^2 + P(u(s))\, ds$$

The minimum of the snake energy satisfies Euler-Lagrange equation [8]:

$$\begin{cases} -\frac{d}{ds}(\alpha u_s(s)) + \frac{d^2}{ds^2}(\beta u_{ss}(s)) + \nabla P(u(s)) = 0 \\ + \text{boundary conditions.} \end{cases} \quad (1)$$

One of the techniques to solve this equation is the Method of Finite Differences (MFD) [8]. Discretizing the snake curve $u_i = (x_i, y_i)$ and approximating derivatives with finite differences, the equation (1) is solved iteratively:

$$\begin{cases} Ax + P_x(x, y) = 0 \\ Ay + P_y(x, y) = 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} x_t = (A + \gamma I)^{-1}(\gamma x_{t-1} - P_x(x_{t-1}, y_{t-1})) \\ y_t = (A + \gamma I)^{-1}(\gamma y_{t-1} - P_y(x_{t-1}, y_{t-1})) \end{cases}$$

The damping parameter γ determines the process convergence rate. The system of equations (2) can be considered as a composition of snake attraction to the potential minimum and shape smoothing in accordance to the internal energy requirements imposed by the stiffness matrix A.

3 Potential of the Snake

3.1 Potential as a logarithmic function of distances

The potential of our snakes is constructed as a function of the distance map because in this way they are able to be attracted both by close and far edge points [3]. Besides the problem of the attraction by far edges, the potential construction is strongly related to the convergence problem. Sliding on the potential field towards its valleys the snake should achieve the edge points without oscilation around them. The simpler way is to directly use the distance map as a potential field. Since the snaxel step is proportional to the potential gradient magnitude, the snake will move due to the external force with constant jumps towards the potential valleys independently from the edges proximity. Another possibility is to smooth the distance map by a Gaussian [4]. The slope is steeper far from the valleys and the snake will jump faster, while approaching the valley the step is reduced in order to not surpass the edge. This approach is useful when the scene does not contain much noise, object texture or different objects' contours. Otherwise, the snake could be more attracted by far edges that often do not correspond to the correct contour, and the close edge points, usually more likely to belong to the correct contour, are ignored. Therefore we generate the potential field for the snake as follows:

$$P(x, y) = log_a(1 + d(x, y)) \quad (2)$$

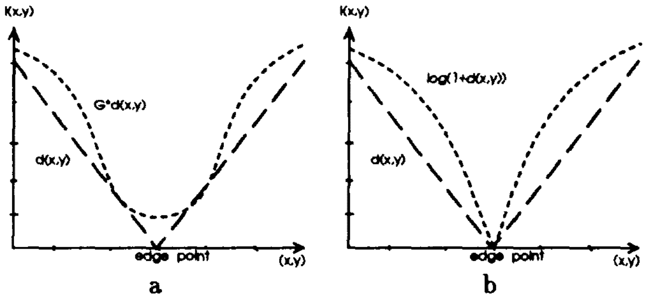

Figure 1: Generation of two kinds of potential fields (see text)

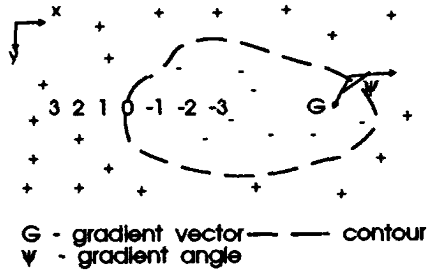

Figure 2: Construction of Signed Distance Potential

where a is a constant dependent on the application and $d(x, y)$ is the distance of pixel (x, y) until the closest edge point.

In Fig.1 a comparison between a potential generated as a distance map smoothed by a Gaussian and as a logarithmic function of the pixel distance to the closest edge point is shown. In the first case the potential gradient in the neighbourhood of the edge point is small, the snake often stops there without falling exactly on the edges. Defining the potential in the second way, the large potential gradient in the neighbourhood of the edge point causes the close snaxel to converge to the edge point. Another property of the so defined potential is the possibility to introduce high-level knowledge by the parameter a. It is referring to what is considered of being close and far edges. This is because the parameter a determines where the slope of the potential function changes significantly, reducing its influence on the snake curve.

3.2 Potential with attributed signs

Though there exist different techniques to generate potentials, in all of them we have information in a point only about the distance and eventually about the gradient magnitude of the closest edge point. Often this information results insufficient. We would save many incorrect displacements if in a certain distance we could obtain some more information about the closest edge in order to decide whether or not to deform the snake towards it. In this sense, such valuable information concerns the edge segment orientation. The potential field of our snake explicitly incorporates information about the gradient direction of the edge points.

The construction of the potential as a distance map is carried out by a modification of the raster scan algorithm [9] exposed in details in [12]. The advantage of this algorithm is its reduced complexity: the number of iteration depends only on the mask size (in our case, 3x3) used to calculate the minimal distance. In order to take into account the gradient direction of the edge points we consider two kinds of potentials: in the first one, the gradient direction is propagated at the same time as the distance. In this case the potential consists of an image with the pixel distances to the closest edge point and an image with the gradient direction of these edge points. The second kind of potential called Signed Distance Potential (SDP) is an image composed by propagated distances, where each distance has attributed sign. We adopted the following rule to assign the attributed sign: the sign is positive when the distance propagation is in opposite direction of the gradient of the closest edge point and negative, otherwise (Fig.2). One early and restricted version of our study on the signed distance potential is reported in [14]. The properties of the potential of the snakes we propose are the following: a) it makes the snake converge to close edges, b) the influence of far edge points is available c) it explicitly incorporates information about the edge orientation and d) there is a possibility to introduce high-level knowledge about what is considered as close and far contours.

The use of the raster scan algorithm has another advantage; it allows the creation in parallel of a family of potentials in accordance to different thresholdings of the edge map [12]. The parallel design of the potential family and the determining adaptable thresholds for the different potentials resulted in significant reducing the computational cost of the snake technique.

4 External Force of the Snake

Usually, the image force is defined by the gradient of the potential field: $F_{ext}(u) = -\nabla P(u)$. However, such an external force does not provide sufficiently dynamic behaviour to the snakes [4]. Different versions of modified external forces appeared in the bibliography claiming to solve some snake problems as snake attraction by far edges and snake shrinking [4].

Our main goal is to design the external force so that correct edge points are the only considered items (i.e. the force is capable of ignoring spurious edge points, e.g. that do not have suitable gradient direction). By a specially defined external force we claim to achieve the following snake properties:

- only edge points with suitable gradient direction should attract the snake, and
- the more similar the normal vector of the snake curve in a given snaxel directed towards the snake interior to the orientation of the gradient vector of the closest edge point is, the more the edge point will attract the snake point.

Let us consider the snake as a closed curve representing the object model contour so that, for each point of the curve we can determine the normal vector $\vec{n}$ directed towards the snake interior. Let ϕ be the

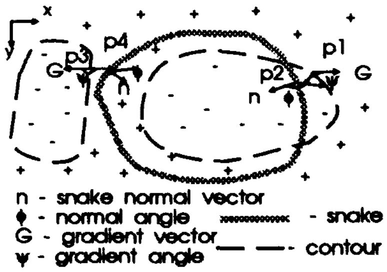

Figure 3: Two examples of pairs of edge points and snaxels with suitable directions (p_1, p_2) and with non-suitable directions (p_3, p_4)

angle formed by this vector with the axis x, and ψ the gradient angle of the closest edge point. The external force defined by the potential gradient $-\nabla P$ makes the snake to be attracted to the 0 pixels of the potential (i.e. the edge points). We consider the following external force:

$$F_{ext} = -(\cos\phi \cos\ \psi P(x,y)_x, \sin\phi \sin\psi P(x,y)_y) \quad (3)$$

The effect of the trigonometric functions is to reduce the snake jump determined by the gradient of the potential. It is easy to see that the image-step-size is maximal when the angles coincide and vice versa: the maximum reducing is achieved when the directions ϕ and ψ are perpendicular. Then the snake is not affected by the external force. For example, in Fig. 3 the vector $\vec{n}$ of the snake pixel p_2 forms angle ϕ similar to the angle ψ formed by the gradient vector G of the closest edge point p_1. Thus, the snake point is attracted by the edge. This is not the case for the pair of snaxel p_4 and its closest edge point p_3. Furthermore, the more similar the angles ϕ and ψ are, the larger the external force module is.

Simplifying the computation of the snake deformation based only on the SDP is useful when we want to accelerate snake calculation process. The external force (3) can be approximated as follows [14]:

$$F_{ext}(x,y) \approx \text{sgn}P(x,y)(\cos\phi P_x(x,y), \sin\phi P_y(x,y)) \quad (4)$$

Defined in this way the external force attracts the snaxels only to edge points with suitable gradient direction. Otherwise, the external force has effect of repulsion. So, we refine the external force to exert only attractive action on the snake:

$$F_1(x,y) = \begin{cases} \text{sgn}(P(x,y))\cos\phi\, P_x(x,y), \\ \quad \text{if } \cos\phi\cos\psi \approx -P_x(x,y)\cos\phi > 0 \\ \quad 0, \quad \text{elsewhere.} \end{cases}$$

$$F_2(x,y) = \begin{cases} \text{sgn}(P(x,y))\sin\phi\, P_y(x,y), \\ \quad \text{if } \sin\phi\sin\psi \approx -P_y(x,y)\sin\phi > 0 \\ \quad 0, \quad \text{elsewhere.} \end{cases}$$

From the definition of the external force it follows that when the closest edge point for a given snake

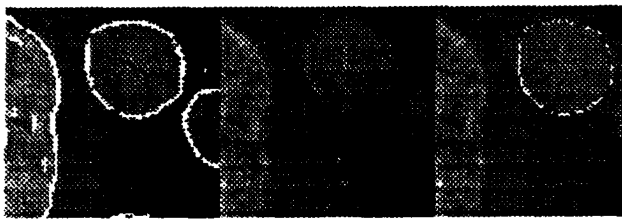

Figure 4: Original image and initial snake (left), result of the classical snake deformation (center), improved result of segmentation by the snakes considered here (right). Edge points are depicted in white.

pixel has not the suitable direction the snake is not affected by the external force. The physical sense of non-attraction corresponds to the cases of hidden contours or non exact snake initialization (the snake is closer to other objects). According to the definition, this part of the snake is affected only by the internal force. If another part of the snake is moved to the correct contour and since the internal force conserves the structure of the object, the former part could be displaced falling in a zone of a correct contour (Fig. 4).

To sum up, the external force enables the snake attraction only by suitable edge points, assures the influence of far edge points and the convergence of the snake by approximating the edges. The external force definition allows to estimate the range of possible values for the snaxel step caused by the image force (something important for the snake convergence and stability) and does not flatten the snake as it happens with the classical snakes in case of e.g. isolated contour segments. The last observation we would like to point out is that the avoidance of spurious edge points does not change the potential with respect to the snake in contrast to other works [13], as the selective snake behaviour makes it independent on certain false edges.

5 Internal Force of the Snake

The model availability suggests us to determine the internal force as follows:

$$F_{int} = \alpha(u_s(s) - u_s^0(s))^2 + \beta(u_{ss}(s) - u_{ss}^0(s))^2 \quad (5)$$

where $u_s^0(s)$ and $u_{ss}^0(s)$ are the derivatives regarding the model contour. Thus a structural information is explicitly incorporated about the desirable shape. The internal force of the snake we consider attempts to compensate the changes caused by the external force and to preserve the object model controlled by the parameters of elasticity α and rigidity β.

Let us develop the mcvement equations of the snake. We obtain:

$$\begin{cases} x_t = (A+\gamma I)^{-1}(\gamma x_{t-1} + Ax^0 + F(x_{t-1}, y_{t-1})) \\ y_t = (A+\gamma I)^{-1}(\gamma y_{t-1} + Ay^0 + F(x_{t-1}, y_{t-1})) \end{cases} \quad (6)$$

Comparing to equations (2) we can see that the difference in the linear equation system for our snakes is in the presence of the members Ax^0 and Ay^0. Hence, we can think about the movement equations of the classic snakes but adding a new force called ***compensating force*** $F_{compens} = (Ax^0, Ay^0)$ aiming to compensate deviations from the initial snake shape. The important property of this force is that it depends only on the initial snake $u^0(s)$. Given a constant stiffness matrix A, the force $F_{compens}$ is constant and calculated only once. The solution of the linear system (6) is determined by the matrix $A + \gamma I$ that is positive definite. This means that the numerical solution has the same properties as those of the classical snake.

One important consequence of the internal force definition is that the force which tries to keep the *model shape, never shrinks the snake*. In the case of constant potential field, the shape is not changed, something more natural than the shrinking in the case of the classical snake. One similar case to the shrinking effect is the accumulation ability of the original snake observed in case of scarce edge segments. Then some parts are accumulated on these segments and the other parts lose their elasticity necessary to expand towards edge points. The accumulation effect is avoided by the use of internal energy to keep the object model.

Using the internal force (6) can be considered as an alternative solution to the problem of the determination of the elasticity parameters. The local dependence of the parameters is necessary when some discontinuities should be allowed in certain parts of the snake. In the case of model-based segmentation, the proximal neigbourhood and the expected kind of discontinuity are available by the structural features of the model. The compensating force explicitly incorporates this information in the snake movement equations. The result is that there is no additional factorization of the stiffness matrix, it is symmetric and positive definite and the computational cost remains low.

6 Application and Results

We have applied our snakes to bone segmentation in hand radiographs. A set of bone models are presented in an atlas [16] that describes the growth process of each bone by means of several maturity stages. Each bone stage is characterized by a typical 2-D shape. Having classified each bone into a maturity stage, it is possible to assess the overall skeletal maturity for an individual. This method is one of the most exact for the skeletal maturity assessment and used in paediatrics to diagnose growth abnormalities. Snakes are very helpful in radiographs segmentation because they are model-directed and offer the possibility of segmenting objects out of discontinuous contours, even in case of absent or hidden parts of the contours, and spurious edge points due to bone texture, noise and near bones.

Our experiments prove the improvement of the snake technique. The behaviour of our snakes has been tested on 20 different bones in 46 images. For each bone we used the models of the two more likely states of maturity as initial models. In 94% the snake we consider detected the correct contour of the bone. For the remaining 6 % the snake solution has only

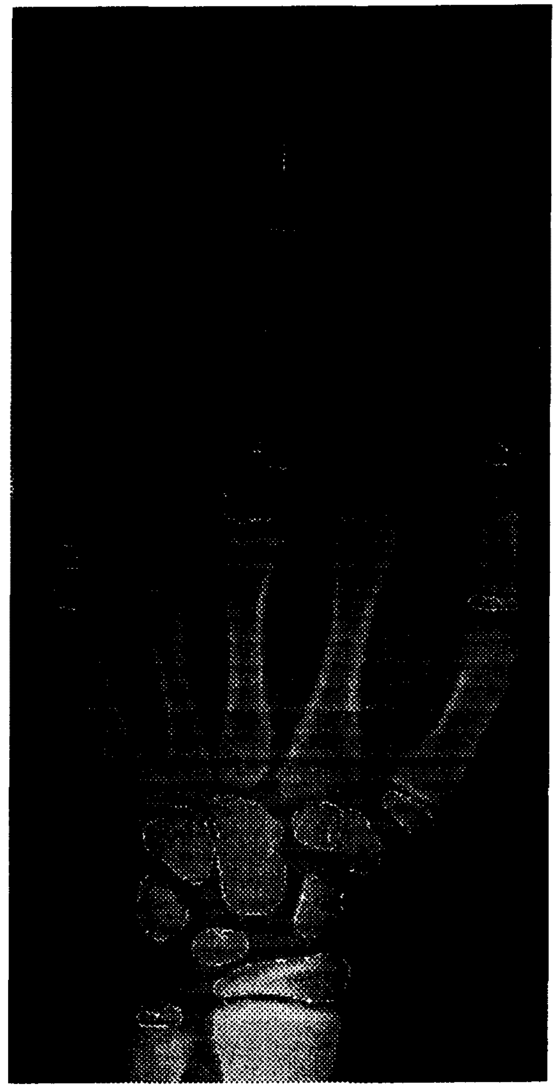

Figure 5: Hand bones segmented by snakes.

some part attracted by some spurious contours (internal borders, internal texture, noise) but no snake result is not satisfactory (most of the bone contour is detected). Segmenting by the original Kass' snakes in the same conditions (same initial snake and initial position) the rate of total success was 30%, in 22% a satisfactory result is obtained (most of the snake curve corresponds to the correct contour) and in 49% no satisfactory result of the snake is observed (most of the snake is attracted by spurious edges).

We explain the difference between the behaviour of both the snakes with the difficult segmentation of the hand bone images (low image contrast, high noise, hidden and absent bone contours, internal borders and texture of the bones, bones overlapping, bones fusion, thickened bone parts, close bones, etc.). Snakes with somewhat "smarter" behaviour had to be implemented in order to achieve satisfactory result. In Fig.

5 we can see a hand radiograph where the bones considered by the TW2 atlas are correctly segmented.

7 Conclusions

Due to current high interest, many variations of snakes were proposed and frequently, the better choice of a specific snake depends on the considered problem. As our goal is to achieve good results they do not exclusively depend on designing new properties of the snakes but also on selecting an optimal combination of different aspects of the available snakes. In our study, we have investigated the different aspects of the snake in order to obtain an optimal behaviour for the case of model-based segmentation.

The snakes we propose implement a new definition of potential field and external energy that solves some problems which affect to all snakes. In many cases, this change avoids snake degeneration, e.g. an oval snake cannot turn into a flat snake. An additional effect is that the snake is not attracted by certain edge points caused by noise or texture. The importance of this technique lies in preventing the deformation towards spurious edge points since the snake can not backtrack. Besides, it is done without eliminating these edge points from the potential image. We show that in the model-based segmentation one good choice is to use internal force that explicitly incorporates structural information about the object we are looking for. This force keeps the properties of the numerical solution of the snake and frees us from the problem of determining local parameters of elasticity and rigidity. Another effect is the prevention of the shrinking, flattening and accumulating effects. The good experimental results obtained foster our investigations and applications in radiograph segmentation. Our present application of snakes is also related to the segmentation of facial features in snapshots, part of our future work on recognition of facial expressions.

Acknowledgements

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