Guidelines for Choosing Optimal Parameters of Elasticity for Snakes *

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Abstract. This paper proposes a guidance in the process of choosing and using the parameters of elasticity of a snake in order to obtain a precise segmentation. A new two step procedure is defined based on upper and lower bounds on the parameters. Formulas, by which these bounds can be calculated for real images where parts of the contour may be missing, are presented. Experiments on segmentation of bone structures in X-ray images have verified the usefulness of the new procedure.

1 Introduction

Snakes have in recent years attracted much attention as an optimal technique for segmentation. The main advantage of this segmentation technique consists in its better performance in cases of noisy images and in scenes consisting of objects with occluded or absent parts of contours [1, 2].

The behaviour of the snake is strongly influenced by its parameters of elasticity. They specify the smoothness of the snake curve. The choice of the parameters is affected by 1) the shape of the object to be segmented, and 2) the image through which the snake moves. As the snake moves it encounters edge fragments caused by noise that may keep it from reaching the contour of the object. For the snake not to be too sensitive to such erroneous edges, emphasis is usually put on the smoothness of its shape. On the other hand, the parameters must allow inherent irregularities of the object. This means putting more emphasis on the actually observed edges in the image. The two constrains on the parameters often turn out contradictory when no formal method is provided to compute them. Interest in determining the parameters of elasticity has been reported [5], but no procedure has been given for their estimation based on a desired smoothness of the detected shape. The choice of parameters are often guided by numerous experiments or by the intuition of a snake expert. We introduce a procedure for choosing the parameters with respect to both constraints by deriving the parameters from the smoothness of a model of the object.

We have previously shown [3, 4] that the upper and lower bounds on the parameters can be calculated based on knowledge about the shape of an object,

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assuming the presence of the entire contour in the image. Here the previous work on perfect contours is extended to handle objects with missing contour fragments. In many cases the bounds outline large intervals for the parameters of elasticity. In this paper we show how the bounds can be used in the procedure for obtaining the parameters optimal for segmentation.

2 Snakes

A snake is an elastic curve, $u(s) = (x(s), y(s))$, for which an energy function E_{snake} is defined based on an external energy E_{ext} and an internal energy E_{int}:

$$E_{snake} = \int_0^1 E_{ext}(u(s)) + E_{int}(u(s))\ ds$$

The external energy is obtained from a potential field derived from the image characteristics. It is typically derived as the image gradient [2] or as a distance map of the edge points [1]. In this work we use the latter definition, i.e. $E_{ext}(u(s)) = P(x, y) = d(x, y)$ where $d(x, y)$ is the distance between pixel (x, y) and its closest edge point.

The internal energy is given by the sum of the membrane energy and the thin-plate energy i.e. $E_{int}(u) = \alpha E_m + \beta E_{tp} = \alpha u'(s) + \beta u''(s)$. The parameters α and β are called parameters of elasticity. α controls the stretching and β the bending of each curve segment.

Minimizing the external energy, the snake is attracted towards the edge points of the image, while minimizing the internal energy the shape of the snake is smoothed. The result of the segmentation by snakes is obtained when the snake detects a minimum of the total energy.

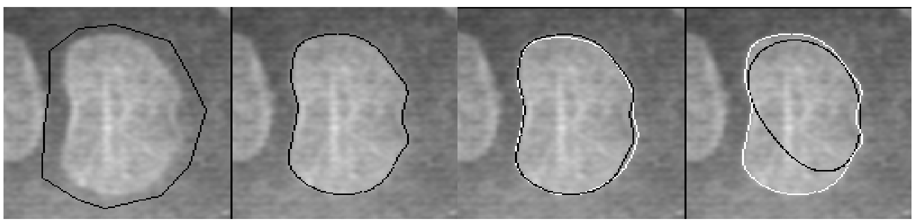
The parameters are weights on the internal energies. If a snake with low parameters, while moving away from the initial position toward the object, meets some erroneous edge segments in the image, the snake finds a local minimum. As the parameter values are increased the snake is able to surpass the erroneous edge, move toward the object and align itself closely around its contour. If the values are increased even more the snake shrinks surpassing the contour of the object. The surpassing of the contour of the object is considered the worst case since it is often related to degeneration of the snake. We start by determining for which values surpassing of the contour can occur.

3 The concept of global and local surpassing

To relate to the problem of surpassing of the contour it is necessary to be able to quantify the amount of surpassing which takes place. Let a contour of an object be represented as a valley in a potential field. Let a closed snake u_0 positioned around the object shrink to a snake u_c positioned in the valley. Consider a snake, u_c, of length n to be placed in the valley. In the following the concepts of global and local surpassing are defined and illustrated as in figure 1.

Definition 1 **Local surpassing** *is when the snake* u_c *in the process of energy minimization moves locally out of the contour valley.* **Global surpassing** *is when the snake* u_c *deforms to a snake outside an* ϵ*-neighbourhood of the contour valley given a constant* ϵ*.*

Definition 2 *The set of maximum values of the elasticity parameters for a snake that does not give rise to any local surpassing is called* **local (surpassing) parameters***. The set of maximum values that, given some* ϵ*, does not give rise to any global surpassing is called* **global (surpassing) parameters***.*



a b c d

Fig. 1. Effect of deformation of the initial snake (a) by optimal (b), local (c) and global (d) parameters. The image shows the contour in white and the snake in black.

The local parameters can be seen as the lower bound on the parameters giving an optimal segmentation. They are the minimum parameters ensuring that the approximating snake has no stronger irregularities than those of the object. The global parameters are the maximum values for which the overall displacement of the snake does not exceed some ϵ-neighbourhood. The neighbourhood is the area in which the snake is influenced by the contour and must be determined by the user. The global parameters determine the upper bound on the parameters.

4 Calculating the bounds

Let us consider a snake u_c positioned in the contour valley and let the snake u_t be its deformation in a distance ϵ. The maximum pair of parameters for which the snake remains in an ϵ-neighbourhood is given by [3, 4],

$$\alpha = \frac{P^t - P^c}{2(E^c_m - E^t_m)}, \quad \beta = \frac{P^t - P^c}{2(E^c_{tp} - E^t_{tp})} \tag{1}$$

where $P^c, E^c_m, E^c_{tp}, P^t, E^t_m, E^t_{tp}$ are the energies of both snakes, respectively.

4.1 Perfect contour

If we have a model of the shape of the object and if the entire contour is present in the image, the global and local surpassing parameters can be calculated. Here only very brief ideas are outlined (for details, see [4]). Without loss of generality all the calculus here are made for a potential field constructed by distance propagation on the edge map. In our FDM snake implementation the distances between pixels are, for the sake of simplicity, kept equal.

In order to estimate the global parameters we substitute in formula (1) the energies of snake u_c placed in the contour valley and the deformed snake u_t after a global surpassing in a distance ϵ. For the case of global parameters we have proposed a way to estimate the different energies of the snake pixels approximating the object contour with a circle of the same length. Thus we obtain,

$$\alpha_{gl} = \frac{n^2}{8\pi(n-\epsilon\pi)}, \qquad \beta_{gl} = \frac{n^4}{32\pi^3(n-\epsilon\pi)}$$

The local parameters are obtained using the energies of the snake u_c and a snake u_t after a local surpassing only in the most curved place of the snake. Substituting into formula (1) the energies for the most curved segment and its deformed version we get,

$$\alpha_{loc} = \frac{\theta\lceil\frac{m}{2}\rceil}{2m(1-2(r+\epsilon)^2(1-\cos\frac{\mu}{m}))}, \qquad \beta_{loc} = \frac{\theta\lceil\frac{m}{2}\rceil r^2}{2m(1-4r^2(r+\epsilon)^2(1-\cos\frac{\mu}{m})^2)}$$

$$\theta = \sqrt{r^2+(r+\epsilon)^2-2r(r+\epsilon)\cos\frac{\nu-\mu}{2}} - \epsilon, \ \mu = \arccos(\frac{\epsilon(2r+\epsilon)+r^2\cos(m\arccos(1-\frac{1}{2r^2}))}{(r+\epsilon)^2})$$

where m, ν and r are the length, the angle and the radius of the most curved segment of the snake u_c. Referring the formulas to the initial snake u_0 we obtain,

$$\alpha_{gl} = \frac{kn^2}{8\pi(n-\epsilon k\pi)} \quad \alpha_{loc} = \frac{\theta\lceil\frac{m}{2}\rceil k^2}{2m(1-2k^2(r+\epsilon)^2(1-\cos\frac{\mu}{m}))}$$

$$\beta_{gl} = \frac{kn^4}{32\pi^3(n-\epsilon k\pi)} \quad \beta_{loc} = \frac{\theta\lceil\frac{m}{2}\rceil r^2k^4}{2m(1-4r^2k^4(r+\epsilon)^2(1-\cos\frac{\mu}{m})^2)}$$

where $k = \frac{u_0}{u_c}$ is the accumulation rate of u_0 shrunk in the contour valley.

4.2 Contour with missing parts

In order to obtain the parameters of elasticity when the contour of the object has some missing parts, the only changes in formula (1) refer to the potential energies of snakes u_c and u_t. We can consider separately the parts of the snake corresponding to the available contour and the parts corresponding to the missing contours. For the former parts the change of the potential energy is as it was shown above. Here we shall consider the change of the potential energy $\overline{P_t} - \overline{P_c}$ of the snake parts $\overline{u_t}$ and $\overline{u_c}$ corresponding to a missing part of the contour.

Given the edge map of the original image, a potential field is constructed propagating distances using a given mask. It results in an image where any edge pixel is zero and any other pixel has a value determined by the formula, $x_{i,j} = \min(x_{i-1,j}+a, x_{i+1,j}+a, x_{i,j-1}+a, x_{i,j+1}+a, x_{i-1,j-1}+b, x_{i-1,j+1}+b, x_{i+1,j-1}+b, x_{i+1,j+1}+b)$.

Let us assume that the contour has one missing part of length m. Due to properties of the distance propagation by mask the value of a pixel in a distance ϵ is: $y = x + (b-a)\min(x,\epsilon) + a\max(0,\epsilon - x)$. Thus we get,

$$\overline{P_t} - \overline{P_c} = \sum_{i=0}^{m}(\overline{P_{i,t}} - \overline{P_{i,c}}) = 2\sum_{i=0}^{\lceil \frac{m}{2} \rceil}(x + (b-a)\min(x,\epsilon) + a\max(0,\epsilon - x) - x) =$$
$$(b-2a)\min{}^2(\epsilon, \lceil \tfrac{m}{2} \rceil) + (2a\epsilon + b - 2a)\min(\epsilon, \lceil \tfrac{m}{2} \rceil) + \epsilon(b-a)\max(0, \lceil \tfrac{m}{2} \rceil - \epsilon).$$

When global parameters are used, the surpassing is supposed to be significant, i.e. $\epsilon \geq \lceil \frac{m}{2} \rceil$ while in case of local parameters $\epsilon < \lceil \frac{m}{2} \rceil$. Using this fact we get,

$$\overline{P_t} - \overline{P_c} = \begin{cases} (b-2a)\lceil \frac{m}{2} \rceil^2 + (2a\epsilon + b - 2a)\lceil \frac{m}{2} \rceil & \text{for global parameters} \\ (2a-b)\epsilon^2 - (2a-b)\epsilon + 2\epsilon(b-a)\lceil \frac{m}{2} \rceil & \text{for local parameters.} \end{cases} \quad (2)$$

From formula (2) it follows that if the number of missing parts increase the displacement of the snake also increase.

Often an expected rate K of missing contours can be estimated from a general information about the image. Then, the contour can have holes with overall length no greater than $K\%n$, where n is the length of the object contour. From formula (2) it is easy to see that for contours with one hole of length m and contours with q holes with overall length m the following relation is valid,

$$\sum_{i=0}^{m}(\overline{P^t} - \overline{P^c}) = \begin{cases} \sum_{j=1}^{q}\sum_{i=0}^{m_j}(\overline{P_j^t} - \overline{P_j^c}) - (j^2 - j)(2a-b)\lceil \frac{m}{2} \rceil^2, & \text{for global parameters} \\ \sum_{j=1}^{q}\sum_{i=0}^{m_j}(\overline{P_j^t} - \overline{P_j^c}) - j\epsilon(\epsilon - 1)(2a-b), & \text{for local parameters.} \end{cases}$$

This shows that the estimated parameters for one missing part of the contour are smaller than the parameters related to the case when the contour has various holes. Therefore, if we use the parameters supposing there is one missing part and in the contour there are more than one, the displacement would have a tendency to be a little larger. We can compensate for the effect of this difference by choosing a smaller displacement ϵ.

Following the line of calculus outlined in section 4.1 updated by the expressions derived for the missing contours case, the following formulas are obtained,

$$\alpha_{gl} = \frac{kn((1-K)\epsilon n + K(b-2a)\lceil \frac{m}{2} \rceil^2 + (2a\epsilon + b - 2a)\lceil \frac{m}{2} \rceil)}{8\epsilon\pi(n - \epsilon k\pi)}$$
$$\beta_{gl} = \frac{kn^3((1-K)\epsilon n + K(b-2a)\lceil \frac{m}{2} \rceil^2 + (2a\epsilon + b - 2a)\lceil \frac{m}{2} \rceil}{32\pi^3(n - \epsilon k\pi)}$$
$$\alpha_{loc} = \frac{k^2((1-K)\theta\lceil \frac{m}{2} \rceil + K(2a-b)\epsilon^2 - (2a-b)\epsilon + 2\epsilon(b-a)\lceil \frac{m}{2} \rceil)}{2m(1 - 2k^2(r+\epsilon)^2(1 - \cos\frac{\mu}{m}))}$$
$$\beta_{loc} = \frac{r^2k^4((1-K)\theta\lceil \frac{m}{2} \rceil + K(2a-b)\epsilon^2 - (2a-b)\epsilon + 2\epsilon(b-a)\lceil \frac{m}{2} \rceil)}{2m(1 - 4r^2k^4(r+\epsilon)^2(1 - \cos\frac{\mu}{m})^2)}$$

5 The procedure for choosing the parameters

In an attempt to characterize how a snake reacts to erroneous edge segments, the following lemma can be stated,

Lemma 1. *Let there be given a contour of an object and let an initial snake be placed around the object. Assume that* (α, β) *are the maximum values for which the snake, once shrunk to the contour valley, remains in its* ϵ*-neighbourhood. For any deformed version of the initial snake in the area determined by the initial snake and the contour, the snake will move outside of its* ϵ*-neighbourhood.*

The Lemma tells us that the initial snake, as it moves toward the object, has the possibility to surpass the erroneous edges with more than ϵ distance and thereby prevent the influence of the edge.

Choosing the parameters for segmentation we want a high value if the initial snake is far from the object. High parameter values allow the snake to surpass the erroneous local minima. In cases where the interior of the object contains erroneous edges, care should be taken not to make the snake surpass too much since once caught by an erroneous edge inside the object, we cannot make the snake move back in the direction of the object contour again.

When the snake is close to the contour it is not likely to be attracted by erroneous edge points and it could benefit from having low parameter values as to better approximate the shape of the object. Values below the local surpassing parameters are not recommended as it can allow extra irregularities.

Using the fact that a difference between the initial snake and the contour is given by the accumulation rate k of the initial snake shrunk to the contour valley, the following formula can be used in calculating the **optimal parameters**,

$$(\alpha_{opt}, \beta_{opt}) = \frac{1}{k}(\alpha_{loc}, \beta_{loc}) + (1 - \frac{1}{k})(\alpha_{gl}, \beta_{gl}) \tag{3}$$

Evidently, when the initial snake has a length similar to the length of the contour ($k \approx 1$), the optimal parameters approximate the local surpassing parameters. When the accumulation rate k is high, ($\frac{1}{k} \rightarrow 0$), the optimal parameters become closer to the global surpassing parameters. We propose the following 2 step procedure for obtaining the best segmentation,

Procedure for choosing the parameters:.

Step 1: Let the initial snake converge with $(\alpha_{opt}, \beta_{opt})$.

Step 2: Converge the snake using $(\alpha_{loc}, \beta_{loc})$.

The first step ensures that the values are high enough to surpass erroneous local energy minima. High values of the parameters which are necessary to move close to the contour, give rise to a local surpassing. Significant improvements in the segmentation are achieved by a new iteration using the local surpassing parameters which brings the snake in more perfect alignment with the contour.

6 Experimental results and discussion

A line of experiments has been conducted on a set of radiographs of bone structures with a significant representation of objects with missing contour parts and additional erroneous edges. In 50 tests on 10 different bone structures the local and global parameters were calculated using different initial snakes and assuming 10% missing contours. 10 different settings were chosen evenly distributed in the intervals between the local and global parameters (table 6). For each of the 10 different parameter settings the snake was iterated and its distance to the contour calculated. The columns 3-13 show the obtained distance between the deformed snake and the object contour. Column 14 shows the best obtained result (the minimum distance) and column 15 gives the distance obtained through parameters of elasticity computed by formula (3).

The effect of the parameters that gave the best segmentation result is quite well approximated by the optimal parameters. This is valid for an initial snake having similar (e.g. $k = 1.28$) and significantly different (e.g. $k = 2.2$) length compared to length of the contour. For the difference between the distances obtained through the optimal and best parameters in our 50 tests, the mean is 0.097 and the variance is 0.022. When there is a difference between the best and the optimal parameter settings (e.g. the last line of the table), the optimal parameters approximate the neighbour setting (in this case, d_6) to the best parameters.

No.	k	d_{gl}	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{loc}	d_{best}	d_{opt}
1	1.28	27.1	27.7	20.9	23.1	16.9	8.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4
2	1.29	27.5	27.3	22.4	24.8	10.6	8.3	0.6	0.6	0.6	0.7	0.7	0.6	0.6
3	1.32	22.6	23.1	22.7	20.7	10.3	0.3	0.3	0.3	0.2	0.3	0.3	0.2	0.2
4	1.34	27.9	22.8	22.5	22.5	11.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
5	1.62	9.2	9.6	14.2	14.2	14.3	3.1	0.3	0.3	3.6	7.6	11.4	0.3	0.3
6	1.73	26.6	25.7	13.5	11.6	9.8	0.2	0.2	0.2	0.2	4.2	12.1	0.2	0.2
7	1.77	26.6	26.7	14.5	14.6	11.3	8.4	0.4	0.4	0.4	0.4	5.1	0.4	0.4
8	1.91	27.2	12.1	12.5	15.6	15.4	0.5	0.4	0.4	0.4	4.4	12.7	0.4	0.4
9	2.2	15.0	15.3	15.7	8.9	4.1	4.1	4.3	0.5	0.5	7.8	17.6	0.5	4.2

Table 1. Distances from the contour in a segmentation by different initial snakes

In case of more missing contours than the expected (e.g. in Fig. 2 where 84% of the contour of the object is available), the optimal parameters are a little larger than the best parameters. Then the snake has some local surpassings (the second image of the second column) in contrast to the snake deformed through the best parameters (the first image from the same column). Since this difference normally is small, applying the local parameters, it is eliminated (the third column of images). When the missing contours are less than the expected, the optimal parameters are smaller than the best ones. This means that the snake with the optimal parameters is not so rigid as the one with the best parameters.

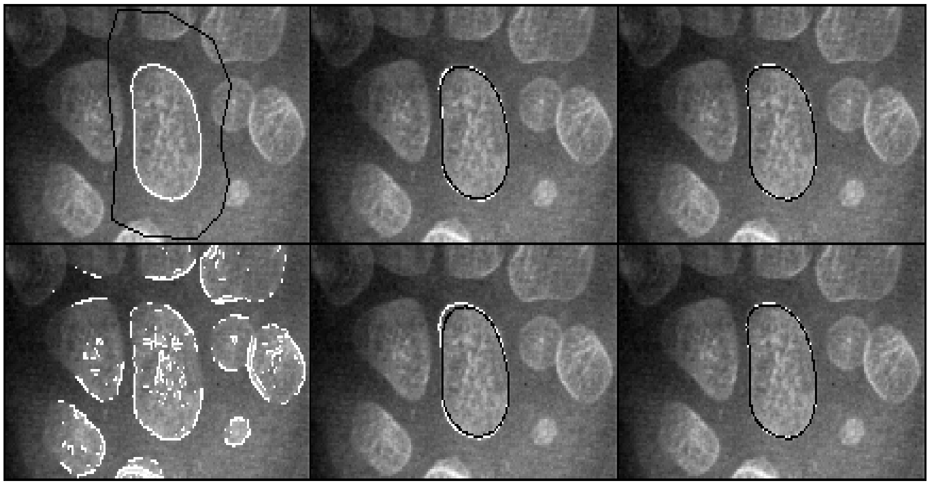


Fig. 2. Different deformations of the initial snake given in black (left top) by best (central top) and optimal (central bottom) parameters. The difference between both deformations is removed by a deformation through the local parameters (right column).

7 Conclusion

Based on previously defined upper and lower bounds on the parameters of elasticity for a snake we have shown how the bounds can be used to formulate a procedure for choosing and using the parameters to provide a precise segmentation. Firstly, parameters close to the upper bound are used to ensure that the initial snake is not attracted by erroneous edges as it iterates towards the object. Secondly, a new setting close to the lower bound can be used to finally ensure a precise segmentation. To calculate the bounds we need a model for the shape of the object of interest to the segmentation. Previous work has calculated the bounds assuming a perfect contour. To address the problem of segmenting objects in noisy images the calculations have been extended to allow for missing contours. Experiments on radiographs of bone structures have provided numerical support for the precision in the segmentation obtained using the procedure.

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