

# A symbol spotting approach in graphical documents by hashing serialized graphs

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## Abstract

In this paper we propose a symbol spotting technique in graphical documents. Graphs are used to represent the documents and a (sub)graph matching technique is used to detect the symbols in them. We propose a graph serialization to reduce the usual computational complexity of graph matching. Serialization of graphs is performed by computing acyclic graph paths between each pair of connected nodes. Graph paths are one dimensional structures of graphs which are less expensive in terms of computation. At the same time they enable robust localization even in the presence of noise and distortion. Indexing in large graph databases involves a computational burden as well. We propose a graph factorization approach to tackle this problem. Factorization is intended to create a unified indexed structure over the database of graphical documents. Once graph paths are extracted, the entire database of graphical documents is indexed in hash tables by locality sensitive hashing (LSH) of shape descriptors of the paths. The hashing data structure aims to execute an approximate  $k$ -NN search in a sub-linear time. We have performed detailed experiments with various datasets of line drawings and compared our method with the state-of-the-art works. The results demonstrate the effectiveness and efficiency of our technique.

*Keywords:* Symbol spotting, Graphics recognition, Graph matching, Graph serialization, Graph factorization, Graph paths, Hashing.

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## 1. Introduction

Even after the significant advancements in the present digital era, paper documents still make an important contribution in our regular work-flows. Digitization of documents is justified on the basis of portability and preservation issues. However, developing a system for browsing and querying digital documents in an effective way still remains a big challenge. So, efficient indexing mechanisms which organize the information extracted by the analysis of document images are essential in order to improve accessibility to these large collections of digital documents. Indexing and retrieval of textual documents involves the conversion of the printed text image into ASCII characters using OCR as the first step. This facilitates the retrieval and querying of information in the document image by textual queries. However nowadays there are some trends to handle textual documents without explicitly recognizing it by OCR [1]. This is either due to reasons of complexity, or all the information in a document can not be represented by typewritten characters. One benefit to be noted about textual data is its single dimensionality that may be sorted, which is not available for graphical objects because of their bi-dimensional nature.

Information spotting is a major branch of indexing and retrieval methods. It can be defined as locating given query information in a large collection of relevant data. In document analysis, the research community is mainly focused on word spotting for textual documents [1, 2] and symbol spotting for graphic-rich documents [3, 4]. Here it is posited that the textual information can also be given a symbolic representation and approached by a symbol spotting technique. In this work we have concentrated on symbol spotting in graphical documents. Architectural line drawings are used as an experimental framework. Symbol spotting can be defined as the identification of a set of regions of interest from document images which are likely to contain an instance of a certain queried symbol using an inexpensive method. Example applications of symbol spotting include finding a mechanical part in a database of engineering drawings or retrieving invoices of a provider from a large database of documents by querying a particular logo. The desired output for a particular query should be a ranked list of retrieved symbols in which the true positives should appear at the beginning. Symbol spotting can be considered a variant of content based image retrieval (CBIR) applications.

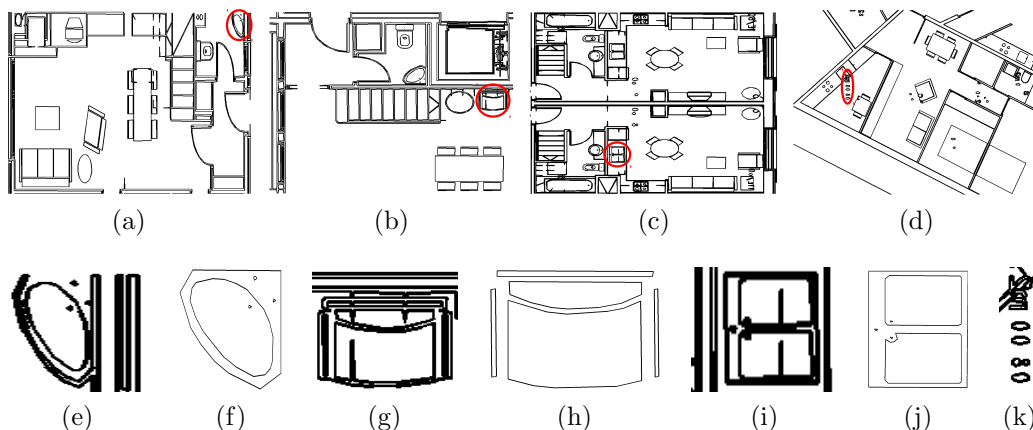


Figure 1: (a)-(d) Examples of floorplans from a real floorplan (FPLAN-POLY) database, (e),(g),(i),(k) Zoomed portions of the selected parts respectively shown in Figures 1a-1d show the difficulty of recognition due to noise and superimposition of textual and graphical information, (f),(h),(j) Actual instances of the symbols shown in (e),(g),(i) respectively.

36 The main differences are that CBIR approaches retrieve the atomic images  
 37 on a large scale leaving the user with the task of locating the real relevant  
 38 information within the provided results, whereas symbol spotting method-  
 39 ologies give direct access to the relevant information. Such applications that  
 40 return direct passages of interests within documents instead of complete doc-  
 41 uments, are known as *focused retrieval* systems [5]. In short, CBIR methods  
 42 can be defined as undertaking image to image matching, whereas focused re-  
 43 trieval is more similar to image to region of interest (ROI) or object location  
 44 searching.

45 Symbol spotting follows the segmentation-recognition paradigm, that is  
 46 a symbol spotting architecture does not use a previous segmentation step  
 47 followed by a proper recognition method, instead it proceeds to coarsely rec-  
 48 ognize and segment in a single step. This demands certain techniques that  
 49 can handle the recognition without segmentation and segmentation without  
 50 recognition at the same time. The problem of symbol spotting in documents  
 51 for real-world situation is more difficult as the documents are arbitrarily ori-  
 52 ented and often suffer from noise (see Figure 1) resulting from scanning,  
 53 vectorization, superimposition of the graphic and textual parts etc. Spot-  
 54 ting methods are usually queried by example i.e. the user segments the item

55 to retrieve from the database and this cropped image acts as input of the  
56 system. This implies infinite possibilities of the query symbols, which pre-  
57 vents explicit training within the spotting architecture. Symbol spotting is  
58 highly applicable for real-time indexing and retrieval from a dataset contain-  
59 ing graphical documents, which demands high efficiency of the method in  
60 terms of computation.

61 Graphs are very suitable data structures to represent graphical docu-  
62 ments, especially line drawings. They allow to capture structural properties  
63 of points, lines, junctions, regions etc. For that reason they have been widely  
64 chosen by the research community as the basic tool to represent graphical  
65 structures [6]. Hence, in line drawings represented by graphs, the problem  
66 of symbol spotting can then be formulated as a subgraph matching problem,  
67 where graph theory offers robust approaches to compute it efficiently. In this  
68 paper we propose a symbol spotting technique based on a graph representa-  
69 tion of graphical documents, especially various kinds of line drawings. When  
70 graphs are attributed by geometric information, this also supports various  
71 affine transformations viz. translation, rotation, scaling etc. In this work,  
72 our representation considers the critical points detected by the vectorization  
73 method [7] as the nodes and the lines joining them as the edges. On the  
74 other hand, subgraph isomorphism is proved to be a NP-hard problem [8],  
75 so handling a large collection of graphical documents using graphs is diffi-  
76 cult. To avoid computational burden, we propose a method based on the  
77 factorization of graphs. Informally, *graph factorization* can be defined as the  
78 method to extract graph sub-structures from larger graphs. This is helpful to  
79 find common subgraph structures from larger collections of graphs to define  
80 indexing keys in terms of such common subgraphs. This indexing structure is  
81 supposed to reduce the search space by clustering similar subgraphs. In our  
82 case, factorization is performed by splitting the graphs into a set of all acyclic  
83 paths (Hamiltonian paths) between each pair of connected nodes. The paths  
84 carry the geometrical information of a structure which are considered as at-  
85 tributes. The decomposition of a graph into graph paths can be seen as a  
86 *serialization* process where the complex two-dimensional graph structure is  
87 converted to a one-dimensional string to reduce computational complexity,  
88 usually present in subgraph matching algorithms. In this work we follow both  
89 factorization and serialization to create an inexpensive and unified structure.  
90 Graph factorization creates a unified representation of the whole database  
91 and at the same time it allows for robust detection with a certain tolerance  
92 to noise and distortion. This also eases the segmentation-free recognition

93 which is important for our purpose.

94 In this work, the shape descriptors of paths are compiled into hash tables  
95 by the Locality-Sensitive Hashing (LSH) algorithm [9, 10]. The hashing data  
96 structure aims to organize similar paths in the same neighborhood into hash  
97 tables. The spotting of the query symbol is then undertaken by a spatial  
98 voting scheme, which is formulated in terms of the selected paths from the  
99 database.

100 However, the graph paths are indexed independently, ignoring any spatial  
101 relationship between them. Actually keeping the spatial relationship is not  
102 important for us since we consider all the acyclic paths between each pair of  
103 connected nodes. Actually this fact better helps to incorporate the structural  
104 noise keeping the spatial relationship among paths. This way the spatial  
105 relationship is maintained as the smaller paths are always subpaths of some  
106 longer paths and longer paths contain more global structural information.

107 Since the method represents a database of graphical documents in terms  
108 of unified representation of factorized substructures, it can handle a larger  
109 database of documents which is important for real-world applications. More-  
110 over, the factorized substructures allow the method to handle structural noise  
111 up to a certain limit of tolerance. The proposed method does not work with  
112 any kind of pre-segmentation and training, which makes it capable of han-  
113 dling any possible combination of query symbols.

114 The rest of the paper is outlined as follows: In Section 2 we survey the  
115 related work in the literated concerning symbol spotting. We present our  
116 proposed methodology in Section 3, followed by a series of experiments in  
117 Section 4. Section 5 concludes the paper with discussions on future works.

## 118 **2. Related work**

119 The focus of this paper is twofold. First, it concentrates on symbol spot-  
120 ting in graphical documents. Second, as a methodological contribution, we  
121 propose an efficient subgraph matching approach based on graph paths. In  
122 this section, we review the literature in both fields.

### 123 *2.1. Symbol spotting*

124 Nowadays symbol spotting has experienced a growing interest among the  
125 graphics recognition community. The major existing research can be classi-  
126 fied into five broad families as in [11], which are listed in Table 1, and those  
127 families are reviewed as follows.

128 *Hidden Markov Models (HMMs)*. HMMs are powerful tools to represent dy-  
129 namic models which vary in terms of time or space. Their major advantage  
130 in space series classification results from their ability to align a pattern along  
131 their states using a probability density function (pdf) for each state, that  
132 estimates the probability of a certain part of the pattern belonging to the  
133 state. HMMs have been successfully applied for off-line handwriting recogni-  
134 tion [12, 13], where the characters represent pattern changes in space whilst  
135 moving from left to right. Also, HMMs have been applied to the problems  
136 of image classification and shape recognition [14]. Müller and Rigoll [15]  
137 proposed pseudo 2-D HMMs to model the two-dimensional arrangements of  
138 symbolic objects. This is one of the first few approaches we can find for sym-  
139 bol spotting, where the document is first partitioned by a fixed sized grid.  
140 Then each small cell acts as an input to a trained 2-dimensional HMM to  
141 identify the locations where the symbols from the model database is likely  
142 to be found. Previously, HMMs were also applied to word spotting, and this  
143 work is an adaptation of HMMs for 2D shapes. The method does not need  
144 pre-segmentation, and also it could be used in noisy or occluded conditions,  
145 but since it depends on the training of a HMM, it loses one of the main  
146 assumptions of symbol spotting methodologies.

147 *Graph-based approaches*. The methods based on graphs rely on the struc-  
148 tural representation of graphical objects and propose (sub)graph matching  
149 techniques to spot symbols in the documents. Graph matching can be solved  
150 with a structural matching approach in the graph domain or solved by a sta-  
151 tistical classifier in the embedded vector space of the graphs. In both cases  
152 these techniques include an error model which allows inexact graph match-  
153 ing to tolerate structural noise in documents. There are an adequate number  
154 of methods based on graphs [16–24]. In general the structural properties of  
155 the graphical entities are encoded in terms of attributed graphs and then a  
156 subgraph matching algorithm is proposed to localize or recognize the sym-  
157 bol in the document in a single step. The (sub)graph matching algorithms  
158 conceive some noise models to incorporate image distortion, which is defined  
159 as inexact (sub)graph matching. Since (sub)graph matching is an NP-hard  
160 problem [8], these algorithms often suffer from a huge computational bur-  
161 den. Among the methods available, Messmer and Bunke in [16] represented  
162 graphic symbols and line drawings by Attributed Relational Graphs (ARG).  
163 Then the recognition process of the drawings was undertaken in terms of  
164 error-tolerant subgraph isomorphisms from the query symbol graph to the

Table 1: Different families of symbol spotting research with their advantages and disadvantages.

Family	Method	Advantages	Disadvantages
HMM	[15]	segmentation-free; Robust in noise	Needs training
Graph based	[16–24]	Simultaneous symbol segmentation and recognition	Computationally expensive
Raster features	[25, 26]	Robust symbol representation; Computationally fast	Ad-hoc selection of regions; Inefficient for binary images
Symbol signatures	[27, 28]	Simple symbol description; Computationally fast	Prone to noise
Hierarchical symbol representation	[29]	Linear matching is avoided by using an indexing technique	Dendogram structure is strongly dependent on the merging criterion.

165 drawing graph. Lladós et al. in [17] proposed Region Adjacency Graphs  
 166 (RAG) to recognize symbols in hand drawn diagrams. They represented the  
 167 regions in the diagrams by polylines where a set of edit operations is defined  
 168 to measure the similarity between the cyclic attributed strings corresponding  
 169 to the polylines. In [18], Barbu et al. presented a method based on frequent  
 170 subgraph discovery with some rules among the discovered subgraphs. Their  
 171 main application is the indexing of different graphical documents based on the  
 172 occurrence of symbols. Qureshi et al. [19] proposed a two-stage method for  
 173 symbol recognition in graphical documents. In the first stage the method only  
 174 creates an attributed graph from the line drawing images and in the second  
 175 stage the graph is used to spot interesting parts of the image that potentially  
 176 correspond to symbols. Then in the recognition phase each of the cropped  
 177 portions from the images are passed to an error tolerant graph matching algo-  
 178 rithm to find the queried symbols. Here the procedure of finding the probable  
 179 regions restricts the method only to work for some specific symbols, which  
 180 violates the assumption of symbol spotting. Locteau et al. [20] present a

181 symbol spotting methodology based on a visibility graph. There they ap-  
182 ply a clique detection method, which corresponds to a perceptual grouping  
183 of primitives to detect regions of particular interest. In [21] Rusiñol et al.  
184 proposed a symbol spotting method based on the decomposition of line draw-  
185 ings into primitives of closed regions. An efficient indexing methodology was  
186 used to organize the attributed strings of primitives. Nayef and Breuel [23]  
187 proposed a branch and bound algorithm for spotting symbols in documents,  
188 where they used geometric primitives as features. Recently Luqman et al.  
189 [22] also proposed a method based on fuzzy graph embedding for symbol  
190 spotting, a priori they also used one pre-segmentation technique as in [19] to  
191 get the probable regions of interest which may contain the graphic symbols.  
192 Subsequently, these ROIs are then converted to fuzzy structural signatures to  
193 find out the regions that contain a symbol similar to the queried one. At last,  
194 very recently, Le Bodic et al. [24] proposed substitution-tolerant subgraph  
195 isomorphism to solve symbol spotting in technical drawings. They represent  
196 the graphical documents with RAG and model the subgraph isomorphism as  
197 an optimization problem. The whole procedure is performed for each pair of  
198 query and document. Moreover, since the method works with RAG, it is not  
199 efficient for the symbols having open regions (for example, Figure 12c,12d)  
200 or regions with discontinuous boundary.

201 *Raster features.* Some of the methods work with low-level pixel features for  
202 spotting symbols. To reduce the computational burden they extract the  
203 feature descriptors on some regions of the documents. These regions may  
204 come from a sliding window or spatial interest point detectors. These kinds  
205 of pixel features robustly represent the region of interest. Apart from those  
206 methods mentioned, other methods find some probable regions for symbols  
207 by examining the loop structures [19] or just use a text/graphic separation  
208 to estimate the occurrence of the symbols [25]. After ad-hoc segmentation,  
209 global pixel-based statistical descriptors [25, 26] are computed at each of  
210 the locations in sequential order and compared with the model symbols. A  
211 distance metric is also used to decide the retrieval ranks and to check whether  
212 the retrievals are relevant or not. The one-to-one feature matching is a clear  
213 limitation of this kind of methods and also the ad-hoc segmentation step only  
214 allows it to work for a limited set of symbols.

215 *Symbol signatures.* Like the previous category, this group of methods [27, 28,  
216 30] also works with ad-hoc segmentation, but instead of pixel features they



217 compute the vectorial signatures, which better represent the structural prop-  
218 erties of the symbolic objects. Here vectorial signatures are the combination  
219 of simple features viz. number of graph nodes, relative lengths of graph edges  
220 etc. These methods are built on the assumptions that the symbols always  
221 fall into a region of interest and compute the vectorial signatures inside those  
222 regions. Since symbol signatures are highly affected by image noise, these  
223 methods do not work well in real-world applications.

224 *Hierarchical symbol representation.* Some of the methods [29] work with the  
225 hierarchical definition of symbols, in which they hierarchically decompose  
226 the symbols and organize the symbols' parts in a network or dendrogram  
227 structure. Mainly, the symbols are split at the junction points and each of  
228 the subparts are described by a proprietary shape descriptor. These subparts  
229 are again merged by a measure of density, building the dendrogram structure.  
230 Then the network structures are traversed in order to find the regions of  
231 interests of the polylines where the query symbol is likely to appear.

232 To conclude the literature review, some of the challenges of symbol spot-  
233 ting can be highlighted from the above state-of-the-art reviews. First, symbol  
234 spotting is concerned with various graphical documents viz. electronic doc-  
235 uments, architectural floorplans etc., which in reality suffer from noise that  
236 may come from various sources such as low-level image processing, interven-  
237 tion of text, etc. So efficiently handling structural noise is crucial for symbol  
238 spotting in documents. Second, an example application of symbol spotting  
239 is to find any symbolic object from a large amount of documents. Hence, the  
240 method should be efficient enough to handle a huge database. Third, sym-  
241 bol spotting is usually invoked by querying a cropping symbol from some  
242 document, which acts as an input query to the system. So it implies infi-  
243 nite possibilities of the query symbols, and indirectly restricts the possibility  
244 of training in the system. Finally, since symbol spotting is related to real-  
245 time applications, the method should have a low computational complexity.  
246 We chose these five important aspects (segmentation, robustness in noise,  
247 training free, computational expenses, robustness with a large database) of  
248 symbol spotting to specify the advantages and disadvantages of the key re-  
249 search, which is listed in Table 2. The above literature review reveals the  
250 lack of solutions for addressing the above challenges altogether. This fact  
251 motivates us to propose a symbol spotting technique which can handle the  
252 above limitations of the existing methods.

Table 2: Comparison of the key works of symbol spotting.

Method	segmentation-free	Robust in noise	Training free	Computationally efficient	Robust with large database
Müller and Rigoll [15]	Yes	Yes	No	Yes	-
Messmer and Bunke [16]	Yes	-	-	No	No
Lladós et al. [17]	Yes	-	Yes	No	No
Barbu et al. [18]	Yes	-	Yes	No	No
Qureshi et al. [19]	No	-	Yes	No	No
Locteau et al. [20]	Yes	No	Yes	Yes	No
Rusiñol et al. [21]	Yes	-	Yes	No	Yes
Rusiñol et al. [31]	Yes	-	Yes	Yes	Yes
Tabbone et al. [25]	No	No	Yes	Yes	-
LeBodic et al. [24]	Yes	No	Yes	No	No
Our method	Yes	Yes	Yes	Yes	Yes

253 *2.2. Graph matching approaches for symbol recognition*

254 In addition to the above state-of-the-art of symbol spotting research, since  
255 our work is concerned with graph representation and matching, we would like  
256 to mention some of the key works in the area of graph matching, which are  
257 very relevant to our work. In general, graph matching has a long list of  
258 methods applied to various kinds of pattern recognition techniques. The in-  
259 terested reader is referred to [6] for more details. In the literature there are  
260 approaches to reduce the computational complexity of graph based methods  
261 and graph serialization is one of them. Serialization aims to reduce the com-  
262 putational complexity of expensive graph matching methods, for that reason  
263 it is often used for many computer vision problems [32–34]. All this re-  
264 search is based on the matching of strings which are often extracted from the  
265 graph representing the images, objects etc. The factorization of graphs into  
266 graph paths creates a one-dimensional structure of complex two-dimensional  
267 graphs and reduces the computational complexity. Originally, the factorized  
268 substructures of graphs are often used to represent bigger graphs in graph  
269 kernels [35]. Even the idea of graph paths is already used as a graph kernel in  
270 [36, 37] and it also simulates the idea of a random walk in a graph structure.

271 The above facts motivate us to work on serialization of graphs.

### 272 **3. Proposed method**

273 Our graph representation considers the critical points detected by the  
274 vectorization method as the nodes and the lines joining them as the edges.  
275 For our purpose we use the vectorization algorithm proposed by Rosin and  
276 West [7]. To avoid the computational burden we propose a method based on  
277 the factorization of graphs. The factorization is performed by splitting the  
278 graphs into a set of all acyclic paths (Hamiltonian paths) between each pair of  
279 connected nodes; the paths carry the geometrical information of a structure  
280 as attributes. The factorization helps to create an unified representation of  
281 the whole database and at the same time it allows robust detection with cer-  
282 tain tolerance to noise and distortion. This also eases the segmentation-free  
283 recognition which is important for our purpose. We have already mentioned  
284 that factorization of graphs is used in kernel based methods and it's princi-  
285 ple motive was to cope with distortions. But the kernel based method can  
286 not utilize the power of indexation which is important for our case as we  
287 concentrate in spotting symbols in bigger datasets efficiently. So indexing  
288 the serialized subgraphical structures is a crucial part for our application.  
289 Our method takes the advantage of the error tolerance as proposed by the  
290 kernel based methods and at the same time the advantage of the indexation  
291 strategy to make the searching efficient. In our work, the shape descriptors  
292 of paths are compiled in hash tables by the Locality-Sensitive Hashing (LSH)  
293 algorithm [9, 10]. The hashing data structure aims to organize similar paths  
294 in the same neighborhood in hash tables and LSH is also proved to perform  
295 an approximate  $k$ -NN search in sub-linear time. The spotting of the query  
296 symbol is then performed by a spatial voting scheme, which is formulated in  
297 terms of the selected paths from the database. This path selection is per-  
298 formed by the approximate search mechanism during the hash table lookup  
299 procedure for the paths that compose the query symbol. The method is  
300 dependent on the overall structure of the paths. This technique is able to  
301 handle the existence of spurious nodes. And since we consider all the acyclic  
302 paths between each pair of connected nodes, the detection or recognition of a  
303 symbol is totally dependent on the overall structure of the majority of paths.  
304 This way the method is able to handle the problem of spurious nodes and  
305 edges. So the introduction of spurious edges and nodes only increases the  
306 computational time in the offline part without hampering the performance.

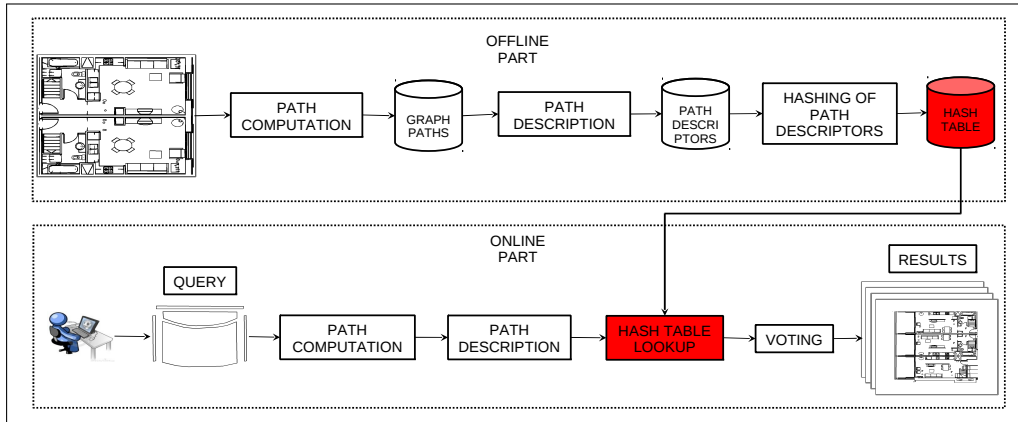


Figure 2: Symbol spotting framework for our method.

307 *3.1. Framework*

308 Our entire framework can be broadly divided into two parts viz. offline  
 309 and online (see Figure 2). The algorithms are respectively shown in Algo-  
 310 rithm 3.1 and Algorithm 3.2. The offline part (Algorithm 3.1) includes the  
 311 computation of all the acyclic graph paths in the database, description of  
 312 those paths with some proprietary descriptors and hashing of those descrip-  
 313 tors using the LSH algorithm (see Figure 3). Each time a new document  
 314 is included in the database, the offline steps for this document are repeated  
 315 to update the hash table. To reduce the time complexity of the offline part  
 316 the path and description information of the previously added documents are  
 317 stored. On the other hand, the online part (Algorithm 3.2) includes the  
 318 querying of the graphic symbol by an end user, the computation of all the  
 319 acyclic paths for that symbol and description of them by the same method.  
 320 Then a hash table lookup for each of the paths in the symbol and a vot-  
 321 ing procedure, which is based on the similarity measure of the paths, are  
 322 also performed on the fly to undertake the spotting in the documents. The  
 323 framework is designed to produce a ranked list of retrievals in which the true  
 324 positive should appear first. The ranking is performed based on the total  
 325 vote values (see subsection 3.4) obtained by each retrieval.

326 Let us now describe the key steps of our framework in the following sub-  
 327 sections.

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328 **Algorithm 3.1** Hash table creation

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329 **Require:** A set  $Doc = \{D_1, \dots, D_n\}$ .

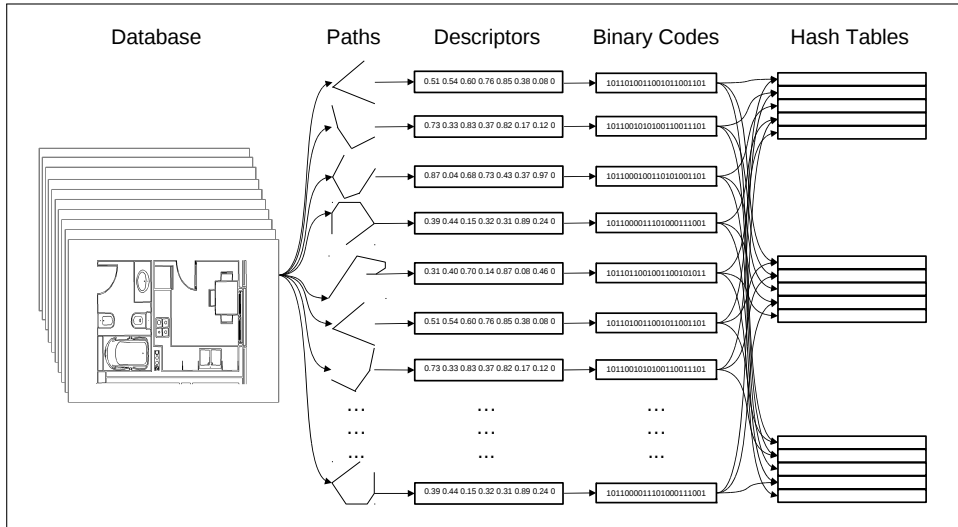


Figure 3: Hashing of paths provokes collisions in hash tables.

```

330 Ensure: A set  $\mathcal{T}$  of hash tables.
331 //Let  $f_{all}$  be the set of all path descriptors.
332 //Initialize  $f_{all}$ 
333  $f_{all} \leftarrow \emptyset$ 
334 for all  $D_i$  of  $Doc$  do
335    $P_i \leftarrow$  acyclic paths ( $D_i$ )
336   for all  $p$  of  $P_i$  do
337      $f \leftarrow$  descriptors of ( $p$ ) // Zernike moments or Hu moment invariants
338      $f_{all} \leftarrow f_{all} \cup f$ 
339   end for
340 end for
341 //Create the set of hash tables
342  $\mathcal{T} \leftarrow$  LSH( $f_{all}$ )

```

### 3.2. Path description

Let  $Doc = \{D_1, D_2, \dots, D_n\}$  be the set of all documents in a database, and  $G_i = (V_i, E_i)$  be the graph for the document  $D_i$ .

**Definition 1.** A graph  $G_i = (V_i, E_i)$  is the ordered pair comprising of the set of vertices  $V_i$  and edges  $E_i$ , here the set  $V_i$  contains all the critical points detected by the vectorization method in a document and  $E_i$  contains the edges joining the vertices in the document.

351 Our graphs are node-labeled and are denoted by  $G_i = (V_i, E_i, L_v)$ .

352 **Definition 2.** Let  $\Sigma$  be the set of all labels for the nodes in a graph. A graph  
 353  $G_i = (V_i, E_i)$  is called node-labeled and denoted as  $G_i = (V_i, E_i, L_v)$ , if there  
 354 is a function  $L_v : V_i \rightarrow \Sigma$ , in this case  $\Sigma = \mathbb{N}^2$ , where the labels for each of  
 355 the nodes is its position in terms of a two-dimensional coordinate system.

356 **Definition 3.** Given a graph  $G_i = (V_i, E_i, L_v)$ , a graph path  $p_k$  between two  
 357 connected nodes  $v_r$  and  $v_s$  in the graph is defined as the ordered sequence of  
 358 vertices  $(v_r, \dots, v_s)$  starting from  $v_r$  to  $v_s$ .

359 **Definition 4.** An embedding function  $f$  of a graph path is defined as a  
 360 function  $f : P \rightarrow \mathbb{R}^n$ , defined in the space of a graph path converts a path to  
 361 an  $n$ -dimensional feature space.

362 Let  $P_i = \{p_1, p_2, \dots, p_{n_i}\}$  be the set of all graph paths in the document  $D_i$ ,  
 363 where  $n_i$  is the total number of paths the document  $D_i$  contains. Therefore  
 364  $P = \cup_i P_i$  is the set of all paths from all the documents in  $Doc$ . From  
 365 the definition of a graph path, a path  $p_k$  can be represented as an ordered  
 366 sequence of nodes i.e.  $p_k = [(x_1, y_1), (x_2, y_2), \dots] = p_k(x, y)$ . So formally  
 367 speaking, given a path  $p_k(x, y)$  and a shape descriptor  $f : P \rightarrow \mathbb{R}^n$  defined  
 368 over the space of all graph paths, applying  $f$  to each of the graph paths in  $P$   
 369 will generate a feature vector of dimension  $n$ . Below is the brief description  
 370 of the shape descriptors used in this work. We define the embedding function  
 371  $f$  by means of Zernike moments and Hu moment invariants.

### 372 3.2.1. Embedding function based on Zernike moments

373 Zernike moments are robust shape descriptors which were first introduced  
 374 in [38] using a set of complex polynomials. They are expressed as  $A_{mn}$  as  
 375 follows:

$$A_{mn} = \frac{m+1}{\pi} \int_x \int_y p_k(x, y) [V_{mn}(x, y)]^* dx dy, \text{ where } x^2 + y^2 \leq 1 \quad (1)$$

376 where  $m = 0, 1, 2, \dots, \infty$  and defines the order,  $p_k(x, y)$  is the path being  
 377 described and  $*$  denotes the complex conjugate. While  $n$  is an integer (that  
 378 can be positive or negative) depicting the angular dependence, or rotation,  
 379 subject to the conditions  $m - |n| = \text{even}$ ,  $|n| \leq m$  and  $A_{mn}^* = A_{m, -n}$  is true.

380 The Zernike polynomials  $V_{mn}(x, y)$  can be expressed in polar coordinates as  
 381 follows:

$$V_{mn}(x, y) = V_{mn}(r, \theta) = \sum_{s=0}^{\frac{m-|n|}{2}} (-1)^s \frac{(m-s)!}{s! \left(\frac{m+|n|}{2} - s\right)! \left(\frac{m-|n|}{2} - s\right)!} \exp(in\theta) \quad (2)$$

382 The final descriptor function  $f_{Zernike}(p_k)$  for  $p_k$  is then constructed by  
 383 concatenating several Zernike coefficients of the polynomials. Zernike mo-  
 384 ments have been widely utilized in pattern or object recognition, image re-  
 385 construction, content-based image retrieval etc. but its direct computation  
 386 takes a large amount of time. Realizing this disadvantage, several algorithms  
 387 [39] have been proposed to speed up the accurate computation process. For  
 388 line drawings, Lambert et al. [40, 41] also formulated Zernike moments as  
 389 computationally efficient line moments. But in our case the computation is  
 390 performed based on the interpolated points of the vectorized data using fast  
 391 accurate calculations.

### 392 3.2.2. Embedding function based on Hu moment invariants

393 The set of seven Hu invariants of moments proposed in [42] involving  
 394 moments up to order three, are widely used as shape descriptors. In general  
 395 the central  $(r + s)$ th order moment for a function  $p_k(x, y)$  is calculated as  
 396 follows:

$$\mu_{rs} = \sum_x \sum_y (x - \bar{x})^r (y - \bar{y})^s \quad (3)$$

397 The function  $f_{Hu}(p_k)$  describing  $p_k$  is then constructed by concatenating  
 398 the seven Hu invariants of the above central moments. The use of centroid  
 399  $c = (\bar{x}, \bar{y})$  allow the descriptor to be translation invariant. A normalization by  
 400 the object area is used to achieve invariance to scale. The geometric moments  
 401 can also be computed on the contour of the objects by only considering the  
 402 pixels of the boundary of the object. As in the case of Zernike moments,  
 403 these moments can also be calculated in terms of line moments [40, 41] for  
 404 the objects represented by vectorized contours, which are obviously efficient  
 405 in terms of computation.

406 *3.3. Locality sensitive hashing (LSH)*

407 In order to avoid one-to-one path matching [43], we use the LSH algorithm  
 408 which performs an approximate  $k$ -NN search that efficiently results in a set  
 409 of candidates that mostly lie in the neighborhood of the query point (path).  
 410 LSH is used to perform contraction of the search space and quick indexation  
 411 of the data. LSH was introduced by Indyk and Motwani [9] and later modified  
 412 by Gionis et al. [10]. It has been proved to perform an approximate  $k$ -  
 413 NN search in sub-linear time and used for many real-time computer vision  
 414 applications.

415 Let  $f(p_k) = (f_1, \dots, f_d) \in \mathbb{R}^d$  be the descriptors of a graph path  $p_k$  in the  
 416  $d$ -dimensional space. This point in the  $d$ -dimensional space is transformed  
 417 in a binary vector space by the following function:

$$v(f(p_k)) = (Unary_C(f_1), \dots, Unary_C(f_d)) \quad (4)$$

418 Here if  $C$  is the highest coordinate value in the path descriptor space  
 419 then  $Unary_C(f_p)$  is a  $|C|$  bit representation function where  $|f_p|$  bits of 1's  
 420 are followed by  $|C - f_p|$  bits of 0's. Thus, the distance between two path  
 421 vectors  $f(p_1)$ ,  $f(p_2)$  can be computed by the Hamming distance between  
 422 their respective binary representations  $v(f(p_1))$ ,  $v(f(p_2))$ . Actually, eqn.(4)  
 423 allows the embedding of the descriptors  $f$ s into the Hamming cube  $H^{d'}$  of  
 424 dimension  $d' = Cd$ . The construction of the function in eqn.(4) assumes the  
 425 positive integer coordinates of  $f$ , but clearly any coordinates can be made  
 426 positive by proper translation in  $\mathbb{R}^d$ . Also the coordinates can be converted  
 427 to an integer by multiplying them with a suitably large number and rounding  
 428 to the nearest integers.

429 Now let  $g : \{0, 1\}^{d'} \rightarrow \{0, 1\}$  be a function which projects a point  $v \in$   
 430  $\{0, 1\}^{d'}$  to any of its  $d'$  coordinate axes, and  $\mathcal{F}$  be a set of such hash functions  
 431  $g(v)$ , which can be formally defined as:

$$\mathcal{F} = \{g(v) | g(v) = v_i, i = 1, \dots, d'\}$$

432 where  $v_i$  is the  $i$ th coordinate of  $v$ . The final set of hash functions  $G$ s  
 433 can be created by randomly selecting at most  $K$  such bitwise hash functions  
 434  $g(v)$  and concatenating them sequentially. This actually results in bucket  
 435 indices in the hash tables. The LSH algorithm then creates a set  $\mathcal{T}$  of  $L$   
 436 hash tables, each of which is constructed based on different  $G$ s.  $L$  and  $K$  are  
 437 considered as the parameters to construct the hashing data structures. Then



438 given a descriptor  $f_q$  of a query path (point), the algorithm iterates over all  
 439 the hash tables in  $\mathcal{T}$  retrieving the data points that are hashed into the same  
 440 bucket. The final list of retrievals is the union of all such matched buckets  
 441 from different hash tables.

442 The entire procedure can be better understood with the following exam-  
 443 ple: let  $f_{p_1} = (1, 6, 5)$ ,  $f_{p_2} = (3, 5, 2)$  and  $f_{p_3} = (2, 4, 3)$  be three different  
 444 descriptors in a three-dimensional ( $d = 3$ ) space with  $C = 6$ . Their binary  
 445 representation after applying the function in eqn. (4) is:

$$\begin{aligned} v(f_{p_1}) &= 100000\ 111111\ 111110 \\ v(f_{p_2}) &= 111000\ 111110\ 110000 \\ v(f_{p_3}) &= 110000\ 111100\ 111000 \end{aligned}$$

446 Now let us create an LSH data structure with  $L = 3$  and  $K = 5$ . So, we  
 447 can randomly create 3 hash functions with at most 5 bits in each of them as  
 448 follows:

$$\begin{aligned} G_1 &= \{g_5, g_{10}, g_{16}\} \\ G_2 &= \{g_1, g_9, g_{14}, g_{15}, g_{17}\} \\ G_3 &= \{g_4, g_8, g_{13}, g_{18}\} \end{aligned}$$

449 This defines which components of the binary vector will be considered to  
 450 create the hash bucket index. For example, applying  $G_2$  to a binary vector  
 451 results in a binary index concatenating the first, ninth, fourteenth, fifteenth  
 452 and seventeenth bit values respectively. After applying the above functions  
 453 to our data we obtain the following bucket indices:

$$\begin{aligned} G_1(f_{p_1}) &= 011, G_2(f_{p_1}) = 11111, G_3(f_{p_1}) = 0110 \\ G_1(f_{p_2}) &= 010, G_2(f_{p_2}) = 11100, G_3(f_{p_2}) = 0110 \\ G_1(f_{p_3}) &= 010, G_2(f_{p_3}) = 11110, G_3(f_{p_3}) = 0110 \end{aligned}$$

454 Then for a query  $f_{p_q} = (3, 4, 5)$  we have

$$\begin{aligned} v(f_{p_q}) &= 111000\ 111100\ 111110 \\ G_1(f_{p_q}) &= 011, G_2(f_{p_q}) = 11111, G_3(f_{p_q}) = 0110 \end{aligned}$$

455 Thus, we obtain  $f_{p_1}$  as the nearest descriptor to the query since it collides  
456 in each of the hash tables.

457 Similarly, for each of the graph path descriptors in the query symbol,  
458 we get a set of paths that belong to the database. Consequently, we get  
459 the similarity distances of the paths in the vectorial space. This similarity  
460 distance is useful during the voting procedure to spot the symbol and is used  
461 to calculate the vote values.

### 462 3.4. Voting scheme

463 A voting space is defined over each of the images in the database divid-  
464 ing them into grids of three different sizes ( $10 \times 10$ ,  $20 \times 20$  and  $30 \times 30$ ).  
465 Multiresolution grids are used to detect the symbols accurately within the  
466 image and the sizes of them are experimentally determined to have the best  
467 performance. It was mentioned earlier that the voting is performed in the  
468 online step of the system when the user query is accepted with a model sym-  
469 bol  $S_m$ . We factorize the graph representing  $S_m$  in the same way as the  
470 documents and let us say  $P_{S_m} = p_1^{S_m}, \dots, p_t^{S_m}$  be the set of all paths of  $S_m$   
471 and  $F_{S_m} = f_{p_1^{S_m}}, \dots, f_{p_t^{S_m}}$  be the set of descriptors for the paths in  $P_{S_m}$ . The  
472 searching in the hash table is then performed in a path by path manner and  
473 consecutively the voting is performed in the image space. For a particular  
474 model path,  $p_l^{S_m} \in P_{S_m}$ , the LSH lookup procedure returns a union of several  
475 buckets (this is how the LSH is constructed). Let us say  $B_l$  be the union  
476 of all buckets returned when queried with a path  $p_l^{S_m}$ . In the next step,  
477 for each path  $f_{p_{B_i}} \in B_l$  we accumulate the votes to the nine neighboring  
478 grids of each of the two terminals of  $f_{p_{B_i}}$  (see Figure 4). The vote to a  
479 particular grid is inversely proportional to the path distance metric (in this  
480 case the Euclidean distance between the Zernike moments descriptors) and  
481 is weighted by the Euclidean distance to the centers of the respective grids  
482 (in Figure 4 the centers of the grids are shown in red) from the terminal of  
483 the selected path. The grids constituting the higher peaks are filtered by  
484 the  $k$ -means algorithm applied in the voting space with  $k=2$ . Here we only  
485 keep the cluster having the higher votes, all the higher voted points from all  
486 the three grids are then considered for spatial clustering. Here we compute  
487 the distances among all these points and use this distance matrix to cluster  
488 the points hierarchically. Here we use a threshold  $th_1$  to cut the dendrogram  
489 and have the clusters. The selection of  $th_1$  is performed experimentally to  
490 give the best performance. Each of the clusters of points is considered as a

491 retrieval; the total vote values of the grids in each cluster are considered for  
 492 ranking the retrievals.

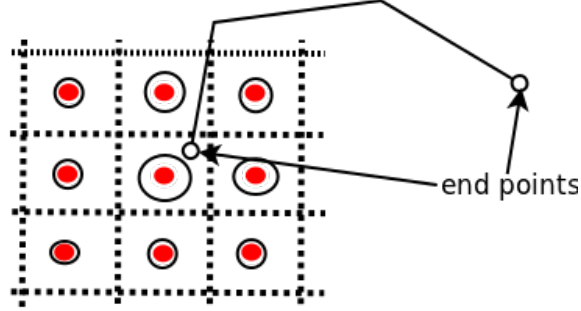


Figure 4: Illustration of voting: For each of the selected paths from the hash table, we accumulate the votes to the nine nearest grids of each of the 2 terminal vertices of that path.

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493 **Algorithm 3.2** Spotting of query symbols in documents

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494 **Require:** A model symbol ( $S_m$ ) with the set of path descriptors  $f_{p_1^{S_m}}, \dots, f_{p_t^{S_m}}$   
 495 and a set  $\mathcal{T}$  of hash tables.  
 496 **Ensure:** A ranked list  $ROI = \{R_1, R_2, \dots\}$  of regions of interest.  
 497 //Search for the nearest buckets  
 498 **for** all  $f_{p_i^{S_m}}$  of  $f_{p_1^{S_m}}, \dots, f_{p_t^{S_m}}$  **do**  
 499  $B_i \leftarrow$  nearest bucket of  $f_{p_i^{S_m}} \in \mathcal{T}$   
 500 //Calculate the matching scores  
 501 **for** all  $f_{p_{B_j}}$  of  $B_i$  **do**  
 502  $MS_{i,j} \leftarrow$  matching score of  $(f_{p_i^{S_m}}, f_{p_{B_j}})$   
 503 **end for**  
 504 **end for**  
 505 //Define and initialize the voting space  
 506 **for** all  $D_k \in Doc$  **do**  
 507 // Grids of three different sizes  
 508 **for** all  $gsize$  of  $\{[10 \times 10], [20 \times 20], [30 \times 30]\}$  **do**  
 509  $G_{gsize}^{D_k} \leftarrow \emptyset$  //Grids on documents  
 510  $GV_{gsize}^{D_k} \leftarrow \emptyset$  //Vote values for the grids  
 511 **end for**  
 512 **end for**

```

513 //Voting
514 for all  $B_i$  of  $B_1, \dots, B_t$  do
515   for all  $f_{p_{B_j}}$  of  $B_i$  do
516      $D \leftarrow$  document of  $f_{p_{B_j}}$ 
517      $[pt_1, pt_2] \leftarrow$  two end points of  $f_{p_{B_j}}$ 
518     for all  $gsize$  of  $\{[10 \times 10], [20 \times 20], [30 \times 30]\}$  do
519       for all  $pt_i$  of  $[pt_1, pt_2]$  do
520          $G_D(1 : 9) \leftarrow$  Nine neighbouring grids of  $pt_i$ 
521          $CG_{gsize}(1 : 9) \leftarrow$  Centres of  $G^D(1 : 9)$ 
522          $GDist(1 : 9) \leftarrow$  distance between  $(CG_{gsize}(1 : 9), pt_i)$ 
523          $GV_{gsize}^D(G_{gsize}^D(1 : 9)) \leftarrow GV_{gsize}^D(G_{gsize}^D(1 : 9)) + GDist(1 : 9) \times$ 
524            $\frac{1}{MS_{i,j}}$ 
525       end for
526     end for
527   end for
528 end for
529 //Spotting
530  $S \leftarrow \emptyset$ 
531 for all  $D_k \in Doc$  do
532   for all  $gsize \in \{[10 \times 10], [20 \times 20], [30 \times 30]\}$  do
533      $[Class_{gsize}^{D_k}(h), Class_{gsize}^{D_k}(l)] = kmeans(GV_{gsize}^{D_k}, 2)$ 
534     //mean( $GV_{gsize}^{D_k}(Class_{gsize}^{D_k}(l)) \leq$  mean( $GV_{gsize}^{D_k}(Class_{gsize}^{D_k}(h))$ ),
535     //where  $GV_{gsize}^{D_k}(Class_{gsize}^{D_k}(h))$  are the higher voted grids
536   end for
537    $G_{all}^{D_k} \leftarrow G_{[10 \times 10]}^{D_k}(Class_{[10 \times 10]}^{D_k}(h)) \cup G_{[20 \times 20]}^{D_k}(Class_{[20 \times 20]}^{D_k}(h)) \cup G_{[30 \times 30]}^{D_k}(Class_{[30 \times 30]}^{D_k}(h))$ 
538    $\{(s_1, total\_votes(s_1)), (s_2, total\_votes(s_2)), \dots\} \leftarrow$  spatial clustering( $G_{all}^{D_k}$ )
539    $S \leftarrow S \cup \{(s_1, total\_votes(s_1)), (s_2, total\_votes(s_2)), \dots\}$ 
540 end for
541  $ROI = sort(S, key = total\_votes)$ 

```

---

542

#### 543 4. Experimental results

544 In this section we present the results of several experiments. The first  
545 experiment is designed to see the efficiency between the Zernike moments  
546 and the Hu moment invariants in a comparative way to represent the graph  
547 paths. The second experiment is undertaken to show the variation of sym-  
548 bol spotting results by varying the  $L$  and  $K$  parameters of the hash table

549 creation. Then a set of experiments is performed to test efficiency of the  
550 proposed method to spot the symbols on documents. For that we use four  
551 different sets of images with varying difficulties. The last experiment is per-  
552 formed to see the possibility of applying the proposed method to any other  
553 information spotting methodologies; for that we test the method with hand-  
554 written word spotting in some real historical handwritten documents. Next  
555 we present a comparative study with a state-of-the-art method. For all these  
556 experiments we mainly use two publicly available databases of architectural  
557 floorplans: FPLAN-POLY<sup>1</sup> [31] and SESYD (floorplans)<sup>2</sup> [44]. The FPLAN-  
558 POLY dataset is a collection of 42 real floorplans (for example see Figure 7a)  
559 and 38 cropped symbols as the queries. The datasets are available in a vec-  
560 torized form and the vectorization is performed by the Qgar<sup>3</sup> software. Con-  
561 versely, the SESYD (floorplans) dataset contains 10 different sub-datasets,  
562 each of which contains 100 different synthetically-generated floorplans (Fig-  
563 ure 7b). All the floorplans in a sub-datasets are created based upon the same  
564 floorplan template by putting different model symbols in different places in  
565 random orientations and scales. Depending upon the need of particular ex-  
566 periments, we introduce some noise models to test the robustness of the  
567 method.

#### 568 4.1. Zernike moments versus Hu moment invariants

569 This test aims to compare the two description methods used to describe  
570 graph paths. Finally, based on this experiment, the best method is used in  
571 the remaining experiments. We compare the performance of the presented  
572 algorithm by using both description methods. To undertake this experiment,  
573 we consider the FPLAN-POLY database and perform the path description  
574 with Hu moment invariants and Zernike moments with different orders (6 to  
575 10). In Figure 5 we show a precision recall curve showing the performance  
576 with different descriptions. This shows that the Zernike moments with any  
577 order outperforms the Hu moment invariants, on average there is a gain of  
578 6.3% precision for a given recall value. Finally, in this experiment, Zernike  
579 moments with order 7 give the best trade-off in terms of performance. This  
580 gives the imperative to perform the rest of the experiments with Zernike  
581 moments descriptors with order 7.

---

<sup>1</sup><http://www.cvc.uab.es/~marcal/FPLAN-POLY/index.html>

<sup>2</sup><http://mathieu.delalandre.free.fr/projects/sesyd/floorplans.html>

<sup>3</sup><http://www.qgar.org/>

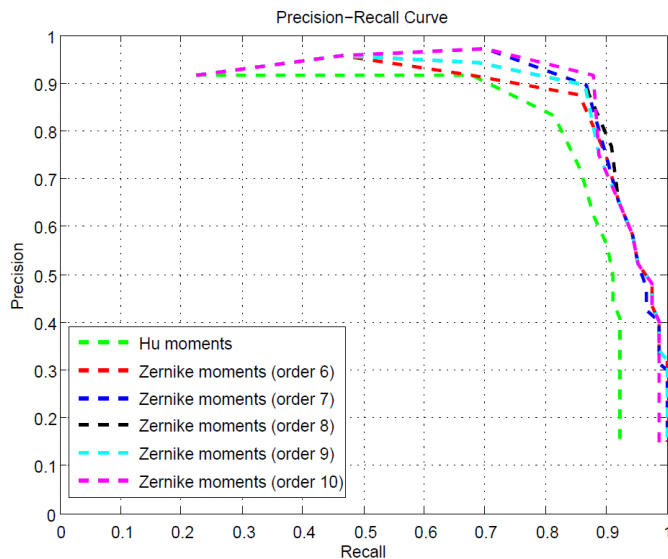


Figure 5: Precision-Recall plot showing the performance of the spotting method with the Hu moment invariants and Zernike moments of order 6 to 10.

#### 582 4.2. Experiments on the influence of parameters $L$ and $K$

583 Literally  $K$  is the maximum number of bits of the binary indices of differ-  
 584 ent buckets in a table. So increasing  $K$  will increase the number of random  
 585 combinations of the bit positions which ultimately increases the number of  
 586 buckets in each of the hash tables. This creates tables in which many buckets  
 587 with only a few instances appear, which separates the search space poorly.  
 588 On the other hand, decreasing  $K$  will merge different instances incorrectly.  
 589 The number of hash tables ( $L$ ) is another parameter to play with, which  
 590 indicates the number of tables to create for a database. Increasing  $L$  will  
 591 increase the search space, since LSH considers the union of all the tables, so  
 592 after a certain limit, increasing the number of tables will not improve the  
 593 performance but only increase the retrieval time. So choosing the proper  
 594 combination of  $L$  and  $K$  for a particular experiment is very important for  
 595 efficient results.

596 In this experiment we chose a set of 10 floorplans from the FPLAN-POLY  
 597 dataset and created the hashing data structures by varying  $L$  from 1 to 20 and  
 598  $K$  from 40 to 80. The performance of the spotting method is shown in terms  
 599 of the precision-recall curves in Figure 6a, which shows similar performance

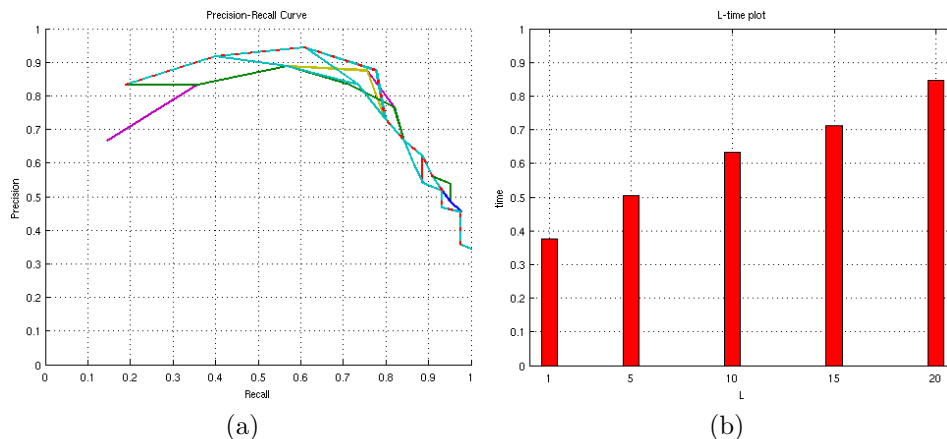


Figure 6: (a) The precision-recall plot of the spotting method by varying  $L$  1 to 20 and  $K$  40 to 80. (b) The plot of the time taken by the method to retrieve symbols for different values of  $L$ .

600 for all the settings. But the time taken by the spotting method increases  
 601 proportionally with the increment of  $L$  (Figure 6b).

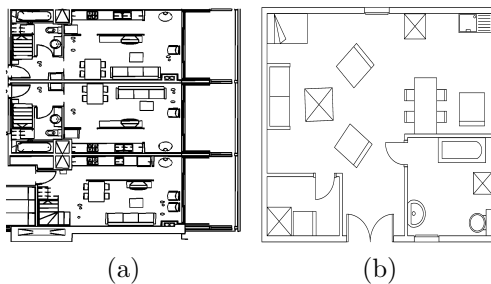


Figure 7: Example images from (a) FPLAN-POLY dataset (b) SESYD dataset.

### 602 4.3. Symbol spotting experiments

603 In order to evaluate the proposed spotting methodology, we present four  
 604 different experiments. The first experiment is designed to test the method  
 605 on the images of real-world floorplans. The second experiment is performed  
 606 to check the algorithm on a moderately large dataset which is a synthe-  
 607 tically created benchmark. Then the experiments are performed to test the  
 608 efficiency of the method on the images of handwritten sketch-like floorplans.



Figure 8: Examples of degraded floorplans. (a)-(c) The same floorplan in Figure 7b degraded with Gaussian noise ( $m=0.1, \sigma=0.01$ ), ( $m=0.3, \sigma=0.05$ ) and ( $m=0.5, \sigma=0.09$ ) respectively, (d)-(f) The same floorplan in Figure 7b degraded with vectorial noise  $r=5, 10$  and  $15$  respectively.

609 Lastly we conducted some experiments to test the method on some noisy  
 610 images, where the kind of noise is very similar to the noise introduced by  
 611 scanning or any other low-level pre-processing.

612 The set of available query symbols for each dataset are used as queries  
 613 to evaluate the ground truths. For each of the symbols, the performance of  
 614 the algorithm is evaluated in terms of precision ( $\mathbf{P}$ ), recall ( $\mathbf{R}$ ) and average  
 615 precision ( $\mathbf{AveP}$ ). In general, the precision ( $\mathbf{P}$ ) and recall ( $\mathbf{R}$ ) are computed  
 616 as:

$$P = \frac{|ret \cap rel|}{|ret|}; R = \frac{|ret \cap rel|}{|rel|} \quad (5)$$

617 Here in eqn. 5, the precision and recall measures are computed on the  
 618 whole set of retrievals returned by the system. That is, they give information  
 619 about the final performance of the system after processing a query and do not  
 620 take into account the quality of ranking in the resulting list. But IR systems  
 621 return results ranked by a confidence value. The first retrieved items are the



622 ones the system believes that are more likely to match the query. As the  
 623 system provides more and more results, the probability to find non-relevant  
 624 items increases. So in this paper the precision value is computed as the  
 625  $P(r_{max})$  i.e. the precision attained at  $r_{max}$ , where  $r_{max}$  is the maximum  
 626 recall attained by the system and average precision is computed as:

$$AveP = \frac{\sum_{n=1}^{n=|ret|} P(n) \times r(n)}{|rel|} \quad (6)$$

627 where  $r(n)$  is an indicator function equal to one, if the item at rank  $n$  is a  
 628 relevant instance or zero otherwise. The interested reader is referred to [45]  
 629 for the definition of the previously mentioned metrics for the symbol spotting  
 630 problem. To examine the computation time we calculate the per document  
 631 retrieval time ( $\mathbf{T}$ ) for each of the symbols. For each of the datasets the mean  
 632 of the above mentioned metrics are shown to judge the overall performance  
 633 of the algorithm.

634 All the experiments described below are performed with the Zernike mo-  
 635 ments descriptors with order 7 (dimension  $d=36$ ). For LSH, the hashing data  
 636 structures are created with  $L=10$  and  $K=60$ . These parameters are experi-  
 637 mentally decided to give the best performance. LSH reduces the search space  
 638 significantly, for example SESYD (floorplans16-01) consists of approximately  
 639 1,465,000 paths and after lookup table construction, these paths are stored  
 640 in 16,000 buckets, so compared to a one-to-one path comparison, the search  
 641 space is reduced by a factor of 90.

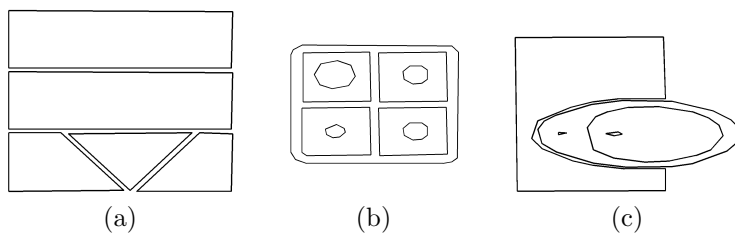


Figure 9: Examples of model symbols from the FPLAN-POLY dataset used for our experiment.

#### 642 4.3.1. Experiment on FPLAN-POLY with real-world images

643 We have tested our method with the FPLAN-POLY dataset. This exper-  
 644 iment is undertaken to show the efficiency of the algorithm on real images,

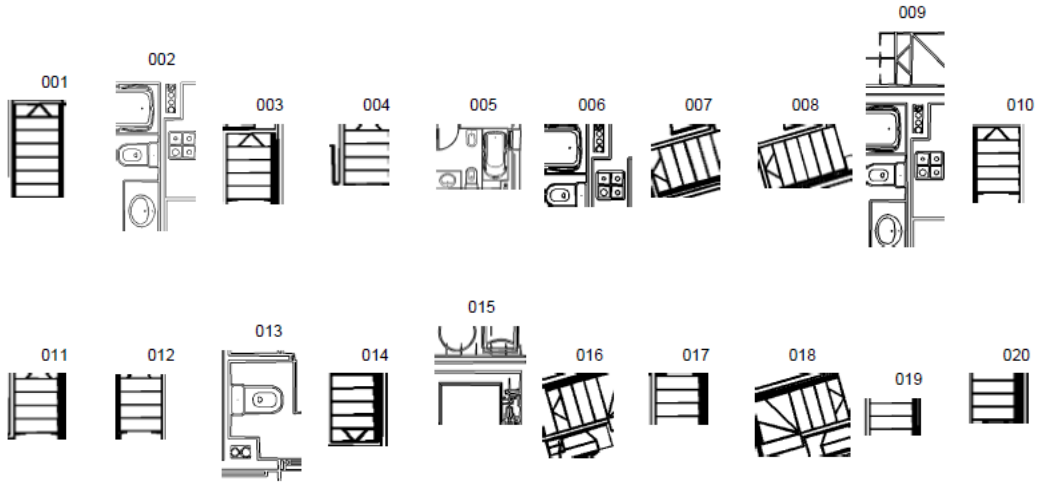


Figure 10: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol in Figure 9a in the FPLAN-POLY dataset.

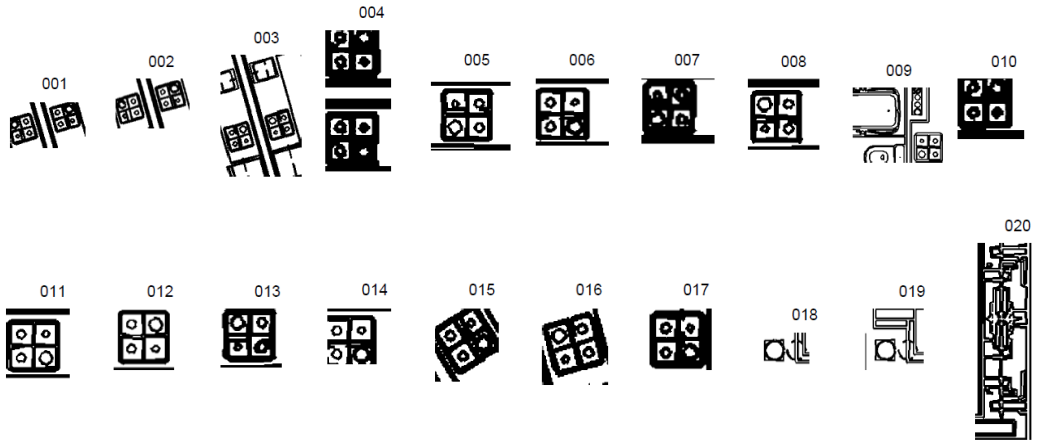


Figure 11: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol in Figure 9b in the FPLAN-POLY dataset.

645 which could suffer from the noise introduced in the scanning process, vector-  
 646 ization etc.

647 The recall rate achieved by the method is 93% which shows the efficiency  
 648 of the algorithm in retrieving the true symbols. The average precision obtained  
 649 by the method is 79.52% which ensures the occupancy of the true  
 650 positives at the beginning of the ranked retrieval list. The precision value

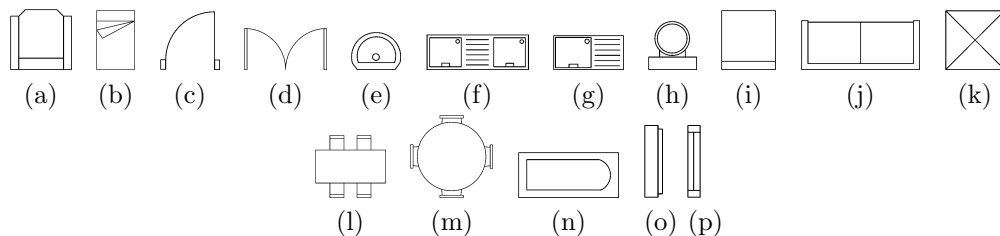


Figure 12: Model symbols in the SESYD dataset.

651 of the method is 77.87% which is more than 50% better than the precision  
 652 reported by the latest state-of-the-art method [21] on this dataset. This sig-  
 653 nifies that the false positives are ranked worse than the correct results. This  
 654 fact is also clear from Figure 10, 11, where we show the qualitative results  
 655 obtained by the method. Also the method is efficient in terms of time com-  
 656 plexity since the average time taken to spot a symbol per document is 0.18  
 657 sec.

#### 658 4.3.2. Scalability experiment on SESYD

659 We have also tested our method on the SESYD (floorplans) dataset. This  
 660 experiment is designed to test the scalability of the algorithm i.e. to check  
 661 the performance of the method on a dataset which is sufficiently large.

Table 3: Results with SESYD dataset

Database	P	R	AveP	T
floorplans16-01	41.33	82.66	52.46	0.07
floorplans16-02	45.27	82.00	56.17	0.09
floorplans16-03	48.75	85.52	71.19	0.07
floorplans16-04	54.51	74.92	65.89	0.05
floorplans16-05	53.25	91.67	67.79	0.08
floorplans16-06	52.70	78.91	60.67	0.07
floorplans16-07	52.78	83.95	65.34	0.07
floorplans16-08	49.74	90.19	58.15	0.08
floorplans16-09	51.92	77.77	47.68	0.07
floorplans16-10	50.96	83.01	63.39	0.08
<b>mean</b>	<b>50.32</b>	<b>83.06</b>	<b>60.87</b>	<b>0.07</b>

662 The mean measurements for each of the sub-datasets are shown in Table

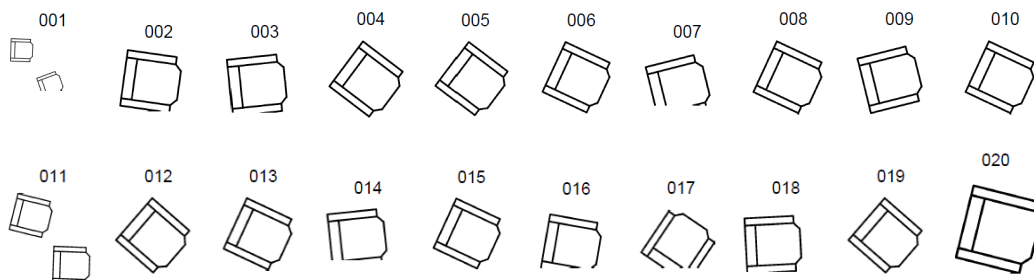


Figure 13: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol shown in Figure 12a in the SESYD (floorplans16-01) dataset.



Figure 14: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol shown in Figure 12d in the SESYD (floorplans16-05) dataset.

663 3. The recall values for all the sub-datasets are quite good, although the  
 664 average precisions are less than in the previous experiments. This is due  
 665 to the existence of the similar substructures (graph paths) among different  
 666 symbols (for example, between the symbols in Figure 12c and Figure 12d  
 667 between the symbols in Figure 12f and Figure 12g, among the symbols in  
 668 Figures 12a, 12b, 12i and 12k and etc). These similarities negatively affect  
 669 the vote values considered for ranking the retrievals. There is an interesting  
 670 observation regarding the average time taken for the retrieval procedure,  
 671 which is 0.07 sec. to retrieve a symbol per document image, which is much  
 672 less than the previous experiment. This is due to the hashing technique,  
 673 which allows for the collision of the same structural elements and inserts  
 674 them into the same buckets. So even though the search space increases due  
 675 to hashing of the graph paths, it remains nearly constant for each of the  
 676 model symbols. This ultimately reduces the per document retrieval time. To  
 677 get an idea about the performance of the method, in Figures 13, 14, 15 and  
 678 16, we present some qualitative results on the SESYD dataset.

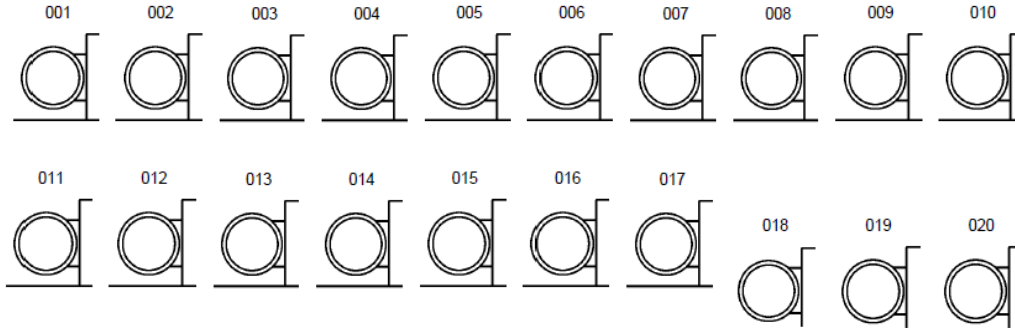


Figure 15: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol shown in Figure 12h in the SESYD (floorplans16-05) dataset.

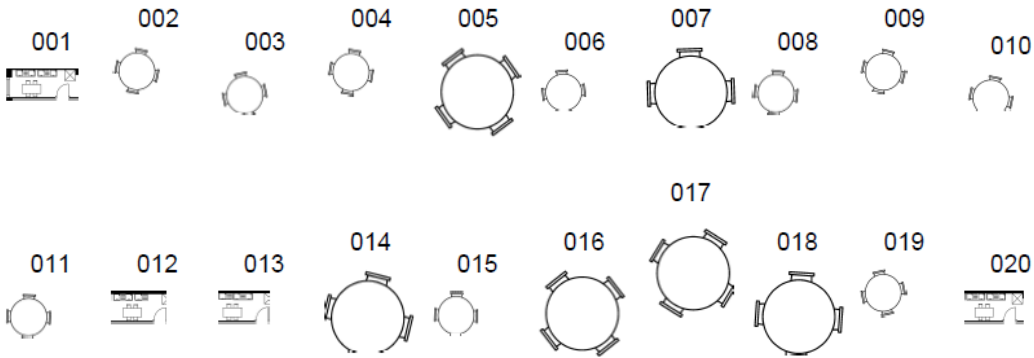


Figure 16: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol shown in Figure 12m in the SESYD (floorplans16-01) dataset.

679 *4.3.3. Experiment on SESYD-VN to test vectorial distortion*

680 This experiment is undertaken to test the effectiveness of the algorithm on  
 681 the handwritten sketch-like floorplans. For this we select one of the 16 sub-  
 682 datasets of the SESYD floorplans and introduces vectorial noise with different  
 683 levels (see Figures 8d, 8e, 8f). The vectorial noise is created by randomly  
 684 shifting the primitive points (critical points detected by the vectorization  
 685 process) within a circle of radius  $r$ . We vary  $r$  to get different level of vectorial  
 686 distortions. For this experiment we have created 3 levels of difficulty (for  
 687  $r = 5, 10, 15$ ). For all the different distortions the same model symbols are  
 688 used as queries.

Table 4: Results with SESYD-VN dataset

<b>Radius (<math>r</math>)</b>	<b>P</b>	<b>R</b>	<b>AveP</b>	<b>T</b>
$r=5$	63.64	92.19	65.27	0.25
$r=10$	47.49	87.01	56.82	0.26
$r=15$	34.37	82.16	47.80	0.25

689 The measurements of the method are shown in Table 4. The recall value  
690 for the dataset with minimum distortion ( $r = 5$ ) is quite good, but it de-  
691 creases with the increment of distortion. The same incident is observed for  
692 average precision also. The distortion also introduces many false positives  
693 which harms the precision. In this experiment, the per document retrieval  
694 time of model symbols increases when compared to the previous experiment.  
695 This is due to the increment of randomness in the factorized graph paths  
696 which decreases the similarity among them. This compels the hashing tech-  
697 nique to create a large number of buckets and hence ultimately increases the  
698 per document retrieval time.

Table 5: Results with SESYD-GN dataset

<b>mean (<math>m</math>)</b>	<b>variance (<math>\sigma</math>)</b>	<b>P</b>	<b>R</b>	<b>AveP</b>	<b>T</b>
0.1	0.01	24.36	94.86	74.07	0.25
	0.05	21.79	89.46	60.07	0.35
	0.09	15.38	67.77	42.85	1.47
0.2	0.01	24.36	94.87	73.43	0.26
	0.05	20.00	82.19	48.93	1.16
	0.09	15.38	65.44	30.97	1.58
0.3	0.01	24.10	93.34	65.79	2.12
	0.05	14.62	69.11	40.81	2.30
	0.09	12.05	54.12	25.62	3.15
0.4	0.01	15.89	72.45	36.32	1.95
	0.05	11.79	50.64	17.97	2.11
	0.09	11.54	43.78	15.29	2.49
0.5	0.01	9.74	34.56	10.00	0.52
	0.05	8.20	29.94	6.69	0.74
	0.09	9.23	36.07	11.14	0.84

699 4.3.4. Experiment on SESYD-GN with noisy images

700 The last symbol spotting experiment is performed to test the efficiency  
 701 of the algorithm on noisy images, which might be generated in the scanning  
 702 process. For this, we also selected one of the 16 sub-datasets of SESYD  
 703 floorplans and introduced Gaussian noise at different levels (see Figure 8a, 8b,  
 704 8c) with the mean ( $m$ ) of 0.1 to 0.5 with step 0.1 and with variance ( $\sigma$ ) 0.01  
 705 to 0.09 with step 0.04, which generates a total 15 sets of images with different  
 706 levels of noise. Practically, the increment of variance introduced more pepper  
 707 noise into the images, whereas the increment of the mean introduced more  
 708 and more white noise, which will detach the object pixel connection. Here  
 709 we do not apply any kind of noise removal technique other than pruning,  
 710 which eliminates isolated sets of pixels.

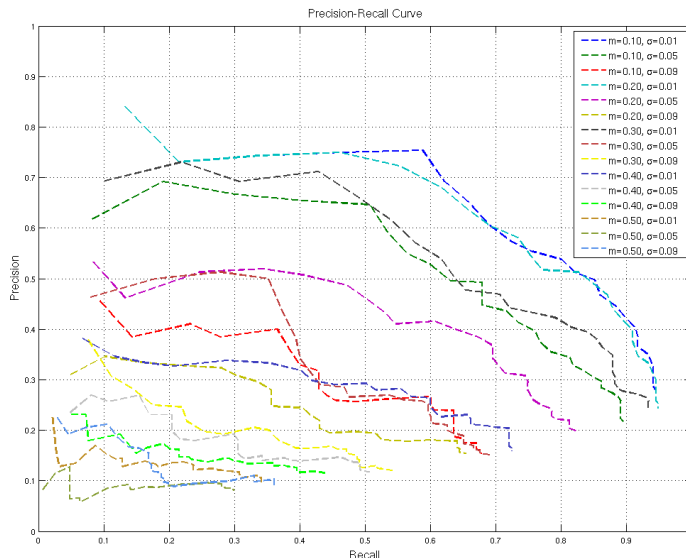


Figure 17: Precision-Recall plot generated by the spotting experiments with different levels of Gaussian noise.

711 The mean measures of metrics are shown in Table 5 and the performance  
 712 of the method is shown in Figure 17 in terms of the precision recall curves.  
 713 Clearly, from the precision-recall curves, the impact of variance is more than  
 714 that of the mean. This implies that with the introduction of more and more  
 715 random black pixels, there is a decrease in the performance, which is due to  
 716 the distortion in the object pixels that substantially affects the vectorization  
 717 methods and changes the local structural features of the graph paths. On

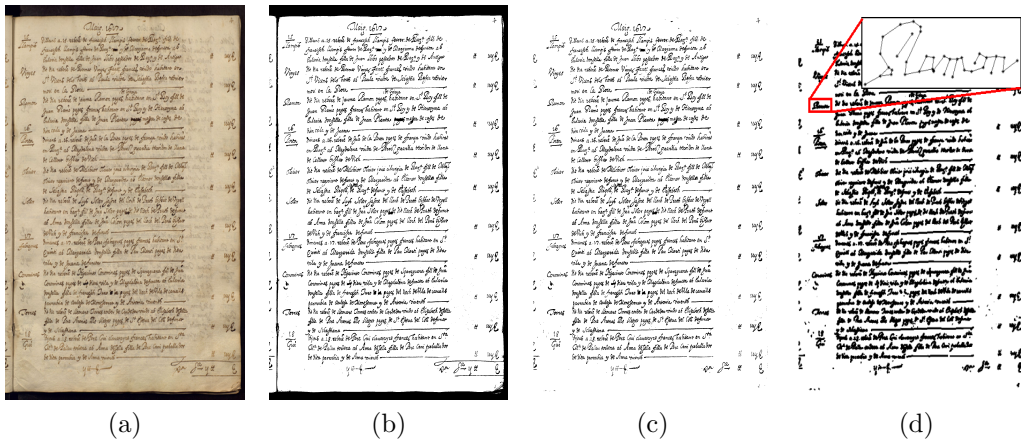


Figure 18: An image from the marriage register from the fifth century from the Barcelona cathedral, (a) The original image, (b) The binarized image of 18a, (c) The image in 18b after preprocessing (eliminating black border created due to scanning), (d) Graph constructed from the image in 18c: the inset also shows the zoomed part of a word 'Ramon'.

718 the other hand, the increment of the mean introduces white pixel noise which  
 719 ultimately separates an object into different parts and which facilitates the  
 720 loss of the local structural information. Increase in Gaussian noise introduces  
 721 local distortions (both with black and white pixels) which introduces extra  
 722 points, as well as discontinuity during the vectorization process. These ran-  
 723 dom points increase the time for computing the paths and also the number  
 724 of buckets due to the random structure of them. Since the increment of the  
 725 mean after a certain stage breaks a component into several pieces, the vector-  
 726 ization results in simple structures of isolated components. These structures  
 727 are quite similar, since in most of the cases they are the straight lines or  
 728 simple combination of straight lines which further decrease the retrieval time  
 729 as they reduce the number of buckets. This explains the increase of retrieval  
 730 time up to a certain stage and then again the decrease. The increment of  
 731 both mean and standard deviation of the Gaussian noise creates a lot of dis-  
 732 continuity within the structure of objects; this creates lot of spurious parts  
 733 after vectorization. These parts are not distinctive among different symbolic  
 734 objects, which explains the irregular shape of the precision recall curves with  
 735 the increase of noise.



#### 736 4.4. *Experiment on handwritten word spotting*

737 This experiment is performed to demonstrate the possibility of apply-  
738 ing our method to any other kind of information spotting system. For that  
739 we have chosen a handwritten word spotting application which also has re-  
740 ceived some popularity amongst the research community. The experiment is  
741 performed on a set of 10 unsegmented handwritten images taken from a col-  
742 lection of historical manuscripts from the marriage register of the Barcelona  
743 cathedral (see Figure 18). Each page of the manuscripts contains approx-  
744 imately 300 words. The original larger dataset is intended for retrieval,  
745 indexing and to store in a digital archive for future access. We use skele-  
746 tonization based vectorization to obtain the vectorized documents. Before  
747 skeletonization, the images undergo preprocessing such as binarization by  
748 Otsu’s method [46] and removal of the black borders generated in the scan-  
749 ning process. Then we construct the graph from the vectorial information  
750 and proceed by considering this as a symbol spotting problem. The retrieval  
751 results of the method on the handwritten images are promising, which is also  
752 clear from the qualitative results shown in Figure 19. This shows a very good  
753 retrieval of the word ”de” with almost perfect segmentation. We also observe  
754 some limitations of the method in spotting handwritten words, among them,  
755 when a particular query word is split into several characters or components,  
756 the method is more prone to retrieve the character, which is more discrim-  
757 inative with respect to the other characters in the word. This is due to  
758 the non-connectivity of the word symbol, which reduces the overall struc-  
759 tural information. Another important observation is that the computation  
760 of paths takes a substantial amount of time for the handwritten documents,  
761 since handwritten characters contain many curves. This generate more and  
762 more spurious critical points in the images, which ultimately affects the path  
763 computation time.

#### 764 4.5. *Discussions*

765 In order to compare the performance of the proposed method with other  
766 methods, we compare our results with three state-of-the-art methods respec-  
767 tively proposed by Luqman et al. [22], Rusiñol et al. [31] and Qureshi et al.  
768 [19]. The method put forward by Luqman et al is based on graph embed-  
769 ding, the method due to Rusiñol et al. is based on the relational indexing of  
770 primitive regions contained in the symbol and that proposed by Qureshi et  
771 al. is based on graph matching, where the methods due to Luqman et al. and  
772 Qureshi et al. [19, 22] use a pre-segmentation technique to find the regions of



Figure 19: The first 120 retrievals of the handwritten word 'de' in the Marriage documents of the Barcelona Cathedral.

773 interest, which probably contain the graphic symbols. Generally this kind of  
 774 localization method works to find some region containing loops and circular  
 775 structures etc. Then a graph matching technique is applied either directly in  
 776 the graph domain or in the embedded space to each of the regions in order  
 777 to match the queried symbol. The method proposed by Rusiñol et al. [31]  
 778 works without any pre-segmentation. For experimentation, we considered  
 779 the images from a sub-dataset of SESYD, The sub-dataset contains 200 im-  
 780 ages of floorplans. The mean measurements at the recall value of 90.00% are  
 781 shown in Table 6 and the performance of the algorithm is shown in terms of  
 782 the precision-recall plot in Figure 20. Clearly, the proposed method domi-  
 783 nates over the existing methods. For any given recall, the precision given by  
 784 our method is approximately 12% more than that reported by Qureshi et al.  
 785 [19], 10% more than that indicated by Rusiñol et al. [31] and 6% more than  
 786 that resulted by Luqman et al. [22], which is a substantial improvement.

787 Finally, we use our algorithm as a distance measuring function between  
 788 a pair of isolated architectural symbols, let us say,  $S_1$  and  $S_2$ . In this case  
 789 we do not perform any hashing, instead we simply factorize the symbols into

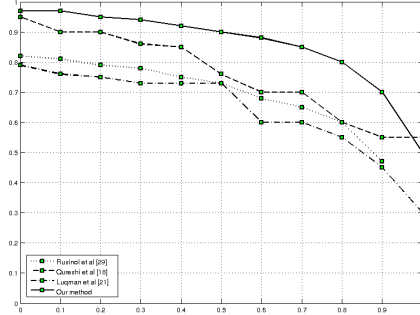


Figure 20: Precision-Recall plot generated by the spotting methods proposed by Luqman et al. [22], Qureshi et al. [19], Rusiñol et al. [31] and our proposed method.

Table 6: Comparison with the state-of-the-art methods

Methods	P	R	AveP	T
Qureshi et al. [19]	45.10	90.00	64.45	1.21
Rusiñol et al. [31]	47.89	90.00	64.51	-
Luqman et al. [22]	56.00	90.00	75.70	-
Our method	70.00	90.00	86.45	0.07

790 graph paths and describe them with some shape descriptors as explained  
 791 in subsection 3.2. Then we use these descriptors to match a path of, say,  
 792 symbol  $S_1$ , to the most identical path of  $S_2$ . So the total distance between  
 793 the symbols  $S_1$  and  $S_2$  is the sum of such distances:

$$\sum_{p_i \in S_1} \min_{p_j \in S_2} dist(p_i, p_j)$$

794 We use this total distance to select the nearest neighbours of the query  
 795 symbol. It is expected that for a pair of identical symbols, the algorithm will  
 796 give a lower distance than for a non-identical symbol. This experiment is  
 797 undertaken to compare our method with various symbol recognition meth-  
 798 ods available in the literature. When using the GREC2005 [47] dataset for  
 799 our experiments, we only considered the set with 150 model symbols. The  
 800 results are summarized in Table 7. We have achieved a 100% recognition  
 801 rate for clear symbols (rotated and scaled) which shows that our method can

802 efficiently handle the variation in scale and rotation. Our method outper-  
803 forms the GREC participants (results obtained from [47]) for degradation  
804 models 1, 2, 3 and 5. The recognition rate decreases drastically for models  
805 4 and 6, this is because the models of degradation lose connectivity among  
806 the foreground pixels. So after the vectorization, the constructed graph can  
807 not represent the complete symbol, which explains the poorer results.

Table 7: Results of symbol recognition experiments

Database	Recognition rate
Clear symbols (rotated & scaled)	100.00
Rotated & degraded (model-1)	96.73
Rotated & degraded (model-2)	98.67
Rotated & degraded (model-3)	97.54
Rotated & degraded (model-4)	31.76
Rotated & degraded (model-5)	95.00
Rotated & degraded (model-6)	28.00

808 In general the symbol spotting results of the system on the SESYD  
809 database are worse than the FPLAN-POLY (see Table 8). This is due to  
810 the existence of more similar symbols in the collection, which often create  
811 confusion amongst the query samples. But the average time for retrieving  
812 the symbols per document is much lower than the FPLAN-POLY database.  
813 This is because of the hashing technique that allows collision of the same  
814 structural elements and inserts them into the same buckets. So even though  
815 the search space increases, due to hashing of the graph paths, it remains  
816 nearly constant for each of the model symbols, which ultimately reduces the  
817 per document retrieval time.

Table 8: Comparative results on two databases FPLAN-POLY & SESYD

Database	P	R	AveP	T
FPLAN-POLY	77.87	93.43	79.52	0.18
SESYD	50.32	83.06	60.87	0.07

818 Our system also produces some erroneous results (see Figures 10(002,  
819 005, 006, 013, 015) and Figures 21(001, 002, 003, 004, 014, 019)) due to  
820 the appearance of similar substructures in nearby locations. For example the

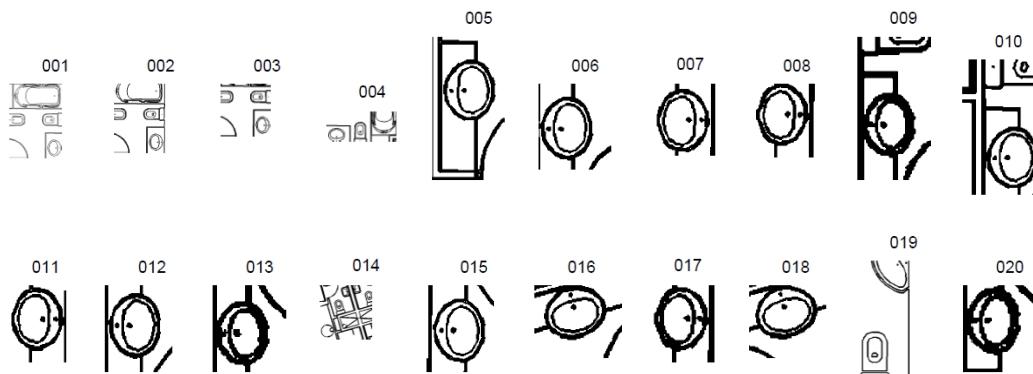


Figure 21: Qualitative results of the method: first 20 retrieved regions obtained by querying the symbol 9c in the FPLAN-POLY dataset.

821 symbol in Figures 9a contains some rectangular box like subparts. The paths  
 822 from these derived substructures of the symbol resemble some commonly  
 823 occurring substructures (walls, mounting boxes etc.) in a floorplan. This  
 824 creates a lot of false votes, which explains the retrieval of the false instances  
 825 in Figure 10. Similarly, the subparts of the symbol in Figure 9c resemble the  
 826 subparts of some architectural symbols. This explains the occurrence of the  
 827 false retrievals in Figure 21.

## 828 5. Conclusions

829 In this paper we have proposed a graph based approach for symbol spot-  
 830 ting in graphical documents. We represent the documents with graphs where  
 831 the critical points detected in the vectorized graphical documents are con-  
 832 sidered as the nodes and the lines joining them are considered as the edges.  
 833 The document database is represented by the unification of the factorized  
 834 substructures of graphs. Here the graph substructures are the acyclic graph  
 835 paths between each pair of connected nodes. The factorized substructures  
 836 are the one-dimensional (sub)graphs which give efficiency in terms of compu-  
 837 tation and since they provide a unified representation over the database, the  
 838 computation is substantially reduced. Moreover, the paths give adaptation  
 839 to some structural errors in documents with a certain degree of tolerance. We  
 840 organize the graph paths in hash tables using the LSH technique, this helps  
 841 to retrieve symbols in real-time. We have tested the method on different  
 842 datasets of various kinds of document images.

843 The main contribution of the method is to deal with a large graph dataset,  
844 whereby dealing with graphs demands more computational complexity. This  
845 has become possible for the factorization technique of graphs which creates  
846 an efficient indexation structure with LSH on top of the database. LSH  
847 makes the organization efficient and the retrieval faster due to the binary  
848 representations of the descriptors. The method has performed quite well in  
849 real-world images, this is also due to the factorization of graphs, which allows  
850 structural noise to a certain level and is very useful for real images. This fact  
851 is also proved when the method performed well with vectorial noise, in this  
852 case of course the performance decreases with the increase of the noise level.  
853 The method performs worse with the increase of Gaussian noise. This kind of  
854 noise introduces lot of spurious points and also disconnections throughout the  
855 vectorization process, which affects the structural attributes of graph paths  
856 and reduces the performance. Also for our experiments we have created some  
857 distorted floorplan datasets represented with graph (SESYD-GN, SESYD-  
858 VN) and we believe the research community will be benefited of the graph  
859 datasets used in the experiments of this paper.

860 The proposed method works with the vectorized information of the docu-  
861 ment and the graph representations are created from vectorized documents.  
862 This implies that the method is highly dependent on the vectorization pro-  
863 cedure. If the vectorization is not robust to noise, even after having some  
864 tolerance to it, the method performs poorly with it, which is clear in the ex-  
865 periment with the pixel (Gaussian) noise. Moreover, when a new document  
866 is included in the database, the system needs to repeat the creation of the  
867 hash table i.e. a part of the offline procedure, which could be considered as  
868 an overhead of the whole system.

869 It is true that the consideration of the graph paths between each pair  
870 of connected nodes creates redundant information but we have argued that  
871 path redundancy is needed to deal the structural noise in the documents.  
872 To reduce the number of redundant paths, we can further think of mutually  
873 exclusive factorization of the graph paths. But this is not straight forward,  
874 moreover, in that case we should take care on the stability of the path struc-  
875 ture. To do that, we can factorize the graphs hierarchically depending on  
876 the curvature of the graph nodes. So, these need further investigations and  
877 experiments which will be our future research issue.

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## 886 **References**

- 887 [1] T. M. Rath, R. Manmatha, Word image matching using dynamic time  
888 warping, in: IEEE International Conference on Computer Vision and  
889 Pattern Recognition, Vol. 2, IEEE Computer Society, Los Alamitos,  
890 CA, USA, 2003, pp. 521–527.
- 891 [2] S. Lu, L. Li, C. L. Tan, Document image retrieval through word shape  
892 coding, IEEE Transactions on Pattern Analysis and Machine Intelli-  
893 gence 30 (2008) 1913–1918.
- 894 [3] J. Lladós, E. Valveny, G. Sánchez, E. Martí, Symbol recognition: Cur-  
895 rent advances and perspectives, in: D. Blostein, Y.-B. Kwon (Eds.),  
896 Graphics Recognition Algorithms and Applications, Vol. 2390 of Lec-  
897 ture Notes in Computer Science, Springer Berlin / Heidelberg, 2002,  
898 pp. 104–128.
- 899 [4] K. Tombre, B. Lamiroy, Pattern recognition methods for querying and  
900 browsing technical documentation, in: 13th Iberoamerican Congress on  
901 Pattern Recognition, CIARP 2008, LNCS, vol. 5197, Springer-Verlag,  
902 2008.
- 903 [5] S. R. Joty, S. Sadid-Al-Hasan, Advances in focused retrieval: A gen-  
904 eral review, in: 10th IEEE International Conference on Computer and  
905 Information Technology (ICCIT 2007), 2007, pp. 1–5.
- 906 [6] D. Conte, P. Foggia, C. Sansone, M. Vento, Thirty years of graph match-  
907 ing in pattern recognition, International Journal of Pattern Recognition  
908 and Artificial Intelligence 18 (3) (2004) 265–298.

- 909 [7] P. L. Rosin, G. A. West, Segmentation of edges into lines and arcs,  
910 Image and Vision Computing 7 (2) (1989) 109 – 114.
- 911 [8] K. Mehlhorn, Graph algorithms and NP-completeness, Springer-Verlag  
912 New York, Inc., New York, NY, USA, 1984.
- 913 [9] P. Indyk, R. Motwani, Approximate nearest neighbors: towards remov-  
914 ing the curse of dimensionality, in: Proceedings of the thirtieth annual  
915 ACM symposium on Theory of computing, STOC '98, ACM, New York,  
916 NY, USA, 1998, pp. 604–613.
- 917 [10] A. Gionis, P. Indyk, R. Motwani, Similarity search in high dimensions  
918 via hashing, in: Proceedings of the 25th International Conference on  
919 Very Large Data Bases, VLDB '99, Morgan Kaufmann Publishers Inc.,  
920 San Francisco, CA, USA, 1999, pp. 518–529.
- 921 [11] M. Rusiñol, Geometric and structural-based symbol spotting. applica-  
922 tion to focused retrieval in graphic document collections, Ph.D. thesis  
923 (2009).
- 924 [12] A. El-Yacoubi, M. Gilloux, R. Sabourin, C. Suen, An hmm-based ap-  
925 proach for off-line unconstrained handwritten word modeling and recog-  
926 nition, Pattern Analysis and Machine Intelligence, IEEE Transactions  
927 on 21 (8) (1999) 752–760.
- 928 [13] S. España-Boquera, M. Castro-Bleda, J. Gorbe-Moya, F. Zamora-  
929 Martinez, Improving offline handwritten text recognition with hybrid  
930 hmm/ann models, IEEE Transactions on Pattern Analysis and Machine  
931 Intelligence 33 (4) (2011) 767–779.
- 932 [14] Y. He, A. Kundu, 2-d shape classification using hidden markov model,  
933 Pattern Analysis and Machine Intelligence, IEEE Transactions on  
934 13 (11) (1991) 1172–1184.
- 935 [15] S. Müller, G. Rigoll, Engineering drawing database retrieval using sta-  
936 tistical pattern spotting techniques, in: Graphics Recognition Recent  
937 Advances, Vol. 1941 of Lecture Notes in Computer Science, Springer  
938 Berlin / Heidelberg, 2000, pp. 246–255.
- 939 [16] B. Messmer, H. Bunke, Automatic learning and recognition of graphical  
940 symbols in engineering drawings, in: R. Kasturi, K. Tomre (Eds.),



- 941 Graphics Recognition Methods and Applications, Vol. 1072 of Lecture  
942 Notes in Computer Science, Springer Berlin / Heidelberg, 1996, pp.  
943 123–134.
- 944 [17] J. Lladós, E. Martí, J. J. Villanueva, Symbol recognition by error-  
945 tolerant subgraph matching between region adjacency graphs, *IEEE*  
946 *Transactions on Pattern Analysis and Machine Intelligence* 23 (2001)  
947 1137–1143.
- 948 [18] E. Barbu, P. Heroux, S. Adam, E. Trupin, Frequent graph discovery:  
949 Application to line drawing document images, *Electronic Letters on*  
950 *Computer Vision and Image Analysis* 5 (2) (2005) 47–54.
- 951 [19] R. Qureshi, J.-Y. Ramel, D. Barret, H. Cardot, Spotting symbols in  
952 line drawing images using graph representations, in: W. Liu, J. Lladós,  
953 J.-M. Ogier (Eds.), *Graphics Recognition. Recent Advances and New*  
954 *Opportunities*, Vol. 5046 of *Lecture Notes in Computer Science*, Springer  
955 Berlin / Heidelberg, 2008, pp. 91–103.
- 956 [20] H. Locteau, S. Adam, Éric Trupin, J. Labiche, P. Héroux, Symbol spot-  
957 ting using full visibility graph representation, in: *Proceedings of 7th*  
958 *International Workshop of Graphics Recognition*, 2007.
- 959 [21] M. Rusiñol, J. Lladós, G. Sánchez, Symbol spotting in vectorized techni-  
960 cal drawings through a lookup table of region strings, *Pattern Analysis*  
961 *and Applications* 13 (2009) 1–11.
- 962 [22] M. Luqman, T. Brouard, J.-Y. Ramel, J. Lladós, A content spotting sys-  
963 tem for line drawing graphic document images, in: *Pattern Recognition*  
964 *(ICPR)*, 2010 20th International Conference on, 2010, pp. 3420–3423.
- 965 [23] N. Nayef, T. M. Breuel, A branch and bound algorithm for graphical  
966 symbol recognition in document images, in: *Proceedings of Ninth IAPR*  
967 *International Workshop on Document Analysis System (DAS, '2010)*,  
968 2010, pp. 543–546.
- 969 [24] P. L. Bodic, P. Héroux, S. Adam, Y. Lecourtier, An integer linear pro-  
970 gram for substitution-tolerant subgraph isomorphism and its use for  
971 symbol spotting in technical drawings, *Pattern Recognition* 45 (12)  
972 (2012) 4214 – 4224.

- 973 [25] S. Tabbone, L. Wendling, K. Tombre, Matching of graphical symbols in  
974 line-drawing images using angular signature information, *International*  
975 *Journal on Document Analysis and Recognition* 6 (2) (2003) 115–125.
- 976 [26] T.-O. Nguyen, S. Tabbone, A. Boucher, A symbol spotting approach  
977 based on the vector model and a visual vocabulary, in: *Document Anal-*  
978 *ysis and Recognition, 2009. ICDAR '09. 10th International Conference*  
979 *on, 2009*, pp. 708–712.
- 980 [27] P. Dosch, J. Lladós, *Vectorial Signatures for Symbol Discrimination*,  
981 Springer Berlin / Heidelberg, 2004, Ch. *Vectorial Signatures for Symbol*  
982 *Discrimination*, pp. 154–165.
- 983 [28] W. Zhang, L. Wenyin, A new vectorial signature for quick symbol index-  
984 ing, filtering and recognition, in: *Proceedings of the Ninth International*  
985 *Conference of Document Analysis and Recognition, Vol. 1, 2007*, pp. 536  
986 –540.
- 987 [29] D. Zuwala, S. Tabbone, *A Method for Symbol Spotting in Graphical*  
988 *Documents*, Springer Berlin / Heidelberg, 2006, pp. 518–528.
- 989 [30] M. Rusiñol, J. Lladós, *Symbol Spotting in Technical Drawings Using*  
990 *Vectorial Signatures*, Springer Berlin / Heidelberg, 2006, Ch. *Symbol*  
991 *Spotting in Technical Drawings Using Vectorial Signatures*, pp. 35–46.
- 992 [31] M. Rusiñol, A. Borràs, J. Lladós, Relational indexing of vectorial prim-  
993 itives for symbol spotting in line-drawing images, *Pattern Recognition*  
994 *Letters* 31 (3) (2010) 188–201.
- 995 [32] J.-M. Jolion, Some experiments on clustering a set of strings, in: E. Han-  
996 cock, M. Vento (Eds.), *Graph Based Representations in Pattern Recog-*  
997 *nition, Vol. 2726 of Lecture Notes in Computer Science*, Springer Berlin  
998 / Heidelberg, 2003, pp. 214–224.
- 999 [33] C. Solnon, J.-M. Jolion, Generalized vs set median strings for histogram-  
1000 based distances: Algorithms and classification results in the image do-  
1001 main, in: F. Escolano, M. Vento (Eds.), *Graph-Based Representations*  
1002 *in Pattern Recognition, Vol. 4538 of Lecture Notes in Computer Science*,  
1003 Springer Berlin / Heidelberg, 2007, pp. 404–414.

- 1004 [34] J. Ros, C. Laurent, J.-M. Jolion, A bag of strings representation for  
1005 image categorization, *Journal of Mathematical Imaging and Vision* 35  
1006 (2009) 51–67.
- 1007 [35] N. Shervashidze, S. V. N. Vishwanathan, T. H. Petri, K. Mehlhorn,  
1008 K. M. Borgwardt, Efficient graphlet kernels for large graph comparison,  
1009 *ReCALL* 5 (2008) 488–495.
- 1010 [36] F.-X. Dupé, L. Brun, *Edition within a Graph Kernel Framework for*  
1011 *Shape Recognition*, Springer-Verlag, Berlin, Heidelberg, 2009, pp. 11–  
1012 20.
- 1013 [37] F.-X. Dupé, L. Brun, *Hierarchical Bag of Paths for Kernel Based Shape*  
1014 *Classification*, Vol. 5342 of *Lecture Notes in Computer Science*, Springer  
1015 Berlin / Heidelberg, 2010, pp. 227–236.
- 1016 [38] M. R. Teague, Image analysis via the general theory of moments, *J. Opt.*  
1017 *Soc. Am.* 70 (8) (1980) 920–930.
- 1018 [39] K. Hosny, Fast computation of accurate zernike moments, *Journal of*  
1019 *Real-Time Image Processing* 3 (2008) 97–107.
- 1020 [40] G. Lambert, H. Gao, Line moments and invariants for real time process-  
1021 ing of vectorized contour data, in: *Image Analysis and Processing*, Vol.  
1022 974 of *Lecture Notes in Computer Science*, Springer Berlin / Heidelberg,  
1023 1995, pp. 347–352.
- 1024 [41] G. Lambert, J. Noll, Discrimination properties of invariants using the  
1025 line moments of vectorized contours, in: *Pattern Recognition, 1996.*,  
1026 *Proceedings of the 13th International Conference on*, Vol. 2, 1996, pp.  
1027 735–739 vol.2.
- 1028 [42] M.-K. Hu, Visual pattern recognition by moment invariants, *Information*  
1029 *Theory*, *IRE Transactions on* 8 (2) (1962) 179–187.
- 1030 [43] A. Dutta, J. Lladós, U. Pal, A bag-of-paths based serialized subgraph  
1031 matching for symbol spotting in line drawings, in: *Proceedings of*  
1032 *5th Iberian Conference on Pattern Recognition and Image Analysis*,  
1033 (IbPRIA2011), Gran Canaria, 2011, pp. 620–627.

- 1034 [44] M. Delalandre, T. Pridmore, E. Valveny, H. Locteau, E. Trupin,  
1035 Building Synthetic Graphical Documents for Performance Evaluation,  
1036 Springer-Verlag, Berlin, Heidelberg, 2008, pp. 288–298.
- 1037 [45] M. Rusiñol, J. Lladós, A performance evaluation protocol for symbol  
1038 spotting systems in terms of recognition and location indices, Interna-  
1039 tional Journal on Document Analysis and Recognition 12 (2) (2009)  
1040 83–96.
- 1041 [46] N. Otsu, A threshold selection method from gray-level histograms, Sys-  
1042 tems, Man and Cybernetics, IEEE Transactions on 9 (1) (1979) 62–66.
- 1043 [47] P. Dosch, E. Valveny, Report on the second symbol recognition contest,  
1044 in: W. Liu, J. Lladós (Eds.), Graphics Recognition. Ten Years Review and  
1045 Future Perspectives, Vol. 3926 of Lecture Notes in Computer Science,  
1046 Springer Berlin / Heidelberg, 2006, pp. 381–397.